**Companion document to the 2018 Mathematics Content Standards**

# Kindergarten Mathematics Content Standards & PLDs

## GRADE K MATH PRACTICES

### MP1 Make sense of problems and persevere in solving them.

**K.MP.1** In Kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students can explain the meaning of a problem and look for ways to solve it. Students check their thinking by using concrete objects or pictures to help them conceptualize and solve problems. Students are also working on increasing stamina as they work on problems.

### MP2 Reason abstractly and quantitatively.

**K.MP.2** Students begin to recognize what a number is and that it also represents a specific quantity. Then, they connect the quantity to written symbols. Students make meaning of word problems and use manipulatives to express and solve their thinking. Students are also working on increasing stamina as they work on problems.

### MP3 Construct viable arguments and critique the reasoning of others.

**K.MP.3** Students construct arguments using concrete illustrations, such as objects, pictures, drawings, and actions. They also begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions such as, “How did you get that? and Why is that true?” They explain their thinking to others and respond to others’ thinking by making connections. Students are also working on increasing stamina as they work on problems.

### MP4 Model with mathematics.

**K.MP.4** Students experiment with representing problem situations in multiple ways including using objects, acting out, drawing pictures, numbers, words (mathematical language), making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.

### MP5 Use appropriate tools strategically.

**K.MP.5** Students begin to explore the different available tools when thinking about the concepts of numbers. They begin to learn which tools help strengthen their understanding of concepts. For instance, kindergarteners may decide that it might be advantageous to use linking cubes to represent two quantities and then compare the two representations side-by-side.

### MP6 Attend to precision.

**K.MP.6** As kindergarteners begin to develop their mathematical communication skills, they try to use clear and precise mathematical vocabulary in their discussions with others and in their own reasoning. Students learn to attend to the shapes of numbers, quickly recognize quantities (subitizing), and simple drawings to show their work.

### MP7 Look for and make use of structure.

**K.MP.7** Students begin to notice a number pattern or structure. For instance, students recognize the pattern that exists in the teen numbers; every teen number is written with a 1 (representing one ten) and ends with the digit that is first stated, and the pattern of numbers 0-9 repeat in the following numbers of $20$, $30$, etc. They also recognize that $3 + 2 = 5$ and $2 + 3 = 5$.

### MP8 Look for and express regularity in repeated reasoning.

**K.MP.8** Students notice repetitive actions in counting and computation, etc. For example, they may notice that the next number in a counting sequence is one more. When counting by tens, the next number in the sequence is ten more (or one more group of ten). Students also notice that when adding two numbers, order of adding doesn’t affect the sum (Commutative Property).

## COUNTING AND CARDINALITY

### Know number names and the count sequence.

**K.CC.A.1**

**K.CC.A.1A Count to 100 by ones and by tens**.

**K.CC.A.1B Count backwards by ones from 20**.

In addition to Proficient, the **Advanced** student is able to:

A1. Count to 100 by ones from a given number.
A2. Count to 100 by tens from a given multiple of ten.
B. Count backward by ones from a given number within 100.

The **Proficient** student is able to:

A1. Count to 100 by ones, starting at one.
A2. Count to 100 by multiples of ten, starting at ten.
B. Count backwards by ones from 20.

The **Basic** student is able to:

A. Count to 100 by ones, with prompting.
B. Count backwards from 10 by ones.

The **Below Basic** student does not meet the Basic performance level.

**K.CC.A.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1)**.

In addition to Proficient, the **Advanced** student is able to count forward a sequence of numbers, crossing two decades/tens starting at a number between 30 and 90.

The **Proficient** student is able to count forward a sequence of 10 numbers, crossing a decade/ten starting at a number between 1 and 30.

The **Basic** student is able to count forward a sequence of 10 numbers, crossing a decade/ten starting at a number between 1 and 30, with prompting.

The **Below Basic** student does not meet the Basic performance level.

**K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with a 0 representing a count of no objects)**.

In addition to Proficient, the **Advanced** student is able to:

A. Write any two digit number above 20.
B. Represent a number of objects with a written numeral between 20 and 40.

The **Proficient** student is able to:

A. Write numbers from 0 to 20.
B. Represent a number of objects with a written numeral 0-20 (with 0 (zero) representing a count of no objects).

The **Basic** student is able to:

A. Write numbers from 0 to 10.
B. Represent a number of objects with a written numeral 0-10 (with 0 (zero) representing a count of no objects).

The **Below Basic** student does not meet the Basic performance level.

## Count to tell the number of objects.

**K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality.**

**K.CC.B.4A** Use one-to-one correspondence when counting objects.

**K.CC.B.4B** Understand that the last number name said, tells the number of objects counted regardless of their arrangement.

**K.CC.B.4C** Understand that each successive number name refers to a quantity that is one more, and each previous number name refers to a quantity that is one less.

In addition to Proficient, the **Advanced** student is able to count up or count back starting at an initial quantity when given an additive or removed item task.

The **Proficient** student is able to count and tell the number of objects in a range from 10 to 39.

A. Use one-to-one correspondence when counting objects.
B. Understand that the last number name said, tells the number of objects counted regardless of their arrangement.
C. Understand that each successive number name refers to a quantity that is one more, and each previous number name refers to a quantity that is one less.

The **Basic** student is able to count and tell the number of objects in a range from 1-9.

A. Use one-to-one correspondence when counting objects.
B. Understand that the last number name said, tells the number of objects counted.

The **Below Basic** student does not meet the Basic performance level.

**K.CC.B.5**

**K.CC.B.5A When counting, answer the question "how many?" by counting up to 20 objects arranged in a line, a rectangular array, a circle, or as many as 10 objects in a scattered configuration.**

**K.CC.B.5B When counting, given a number from 1-20, count out that many objects.**

In addition to Proficient, the **Advanced** student is able to, when counting:

A. Answer the question "how many?" by counting beyond 20 in a scattered configuration.
B. Count out the number of objects given a number 25-35.

The **Proficient** student is able to, when counting:

A. Answer the question "how many?" by counting up to 20 objects arranged in a line, a rectangular array, a circle, or as many as 10 objects in a scattered configuration.
B. Count out the number of objects given a number from 1-20.

The **Basic** student is able to, when counting:

A. Answer the question "how many?" by counting up to 20 objects arranged in a line, a rectangular array, a circle, or as many as 10 objects in a scattered configuration. **OR**
B. Count out the number of objects given a number from 1-20.

The **Below Basic** student does not meet the Basic performance level.

### Compare numbers.

**K.CC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects.)**

In addition to Proficient, the **Advanced** student is able to order 3 or more groups of objects (Include groups with up to ten objects.) from greatest to least or least to greatest.
The **Proficient** student is able to identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group. (Include groups with up to ten objects.)
The **Basic** student is able to identify whether the number of objects in one group is equal to or not equal to the number of objects in another group. (Include groups with up to five objects.)
The **Below Basic** student does not meet the Basic performance level.

**K.CC.C.7 Compare two numbers between 1 and 10 presented as written numerals.**

In addition to Proficient, the **Advanced** student is able to compare 3 or more non-consecutive numbers between 1 and 20 presented as written numerals.
The **Proficient** student is able to compare two numbers between 1 and 10 presented as written numerals.
The **Basic** student is able to compare two numbers between 1 and 5 presented as written numerals.
The **Below Basic** student does not meet the Basic performance level.

## OPERATIONS AND ALGEBRAIC THINKING

### Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

**K.OA.D1 Model situations that involve representing addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.**

In addition to Proficient, the **Advanced** student is able to model situations that involve representing addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, and write the corresponding expression, or equation.
The **Proficient** student is able to model situations that involve representing addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
The **Basic** student is able to model situations that involve representing addition with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
The **Below Basic** student does not meet the Basic performance level.

**K.OA.D2 Solve word problems using objects and drawings to find sums up to 10 and differences within 10.**

In addition to Proficient, the **Advanced** student is able to create a word problem to find sums up to 10 and differences within 10.
The **Proficient** student is able to solve word problems using objects and drawings to find sums up to 10 and differences within 10.
The **Basic** student is able to solve word problems using objects and drawings to find sums up to 5 and differences within 5.
The **Below Basic** student does not meet the Basic performance level.

**K.OA.D3 Decompose numbers less than or equal to 10 in more than one way.**

In addition to Proficient, the **Advanced** student is able to decompose numbers with parts that are less than or equal to 10 in more than one way identifying patterns.
The **Proficient** student is able to decompose numbers less than or equal to 10 in more than one way.

The **Basic** student is able to decompose numbers less than or equal to 5 in more than one way.
The **Below Basic** student does not meet the Basic performance level.

**K.OA.D4 For any number from 1 to 9, find the number that makes 10 when added to the given number.**
In addition to Proficient, the **Advanced** student is able to, for any number from 10-19, find the number that makes 20 when added to the given number.
The **Proficient** student is able to for any number from 1 to 9, find the number that makes 10 when added to the given number.
The **Basic** student is able to for any number from 1 to 4, find the number that makes 5 when added to the given number.
The **Below Basic** student does not meet the Basic performance level.

**K.OA.D5 Fluently add and subtract within 5.**
In addition to Proficient, the **Advanced** student is able to fluently add and subtract within 5 including missing addend problems.
The **Proficient** student is able to fluently add and subtract within 5.
The **Basic** student is able to fluently add within 5.
The **Below Basic** student does not meet the Basic performance level.

## NUMBERS AND OPERATIONS IN BASE TEN

### Work with numbers 11-19 to gain foundations for place value.

**K.NBT.E.1**

**K.NBT.E.1A Describe, explore, and explain how the counting numbers 11 to 19 are composed of ten ones and more ones.**
**K.NBT.E.1B Describe, explore, and explain how the counting numbers 11 to 19 are decomposed into ten ones and more ones.**
In addition to Proficient, the **Advanced** student is able to describe, explore, and explain how the counting numbers 11 to 19 are:
A. Composed of one unit of ten and more ones.
B. Decomposed into one unit of ten and more ones.
The **Proficient** student is able to describe, explore, and explain how the counting numbers 11 to 19 are:
A. Composed of ten ones and more ones.
B. Decomposed into ten ones and more ones.
The **Basic** student is able to describe, explore, and explain how the counting numbers 11 to 19 are composed of ten ones and more ones.
The **Below Basic** student does not meet the Basic performance level.

## MEASUREMENT AND DATA

### Describe and compare measurable attributes.

**K.MD.F.1 Describe several measurable attributes of one or more objects.**
In addition to Proficient, the **Advanced** student is able to describe several measurable and non-measurable attributes of one or more real-world object(s).
The **Proficient** student is able to describe several measurable attributes of one or more objects.
The **Basic** student is able to describe at least two attributes of one or more objects.
The **Below Basic** student does not meet the Basic performance level.

**K.MD.F.2 Make direct comparisons of the length, capacity, weight, and temperature of objects, and recognize which object is shorter/longer, taller, lighter/heavier, warmer/cooler, and which holds more/less.**

In addition to Proficient, the **Advanced** student is able to make a direct comparison of three or more objects based on the length, capacity, weight, and temperature of objects, and order them shorter/longer, taller, lighter/heavier, warmer/cooler, and which holds more/less and vice versa.
The **Proficient** student is able to make direct comparisons of the length, capacity, weight, and temperature of objects, and recognize which object is shorter/longer, taller, lighter/heavier, warmer/cooler, and which holds more/less.
The **Basic** student is able to make direct comparisons of the length of objects, and recognize which object is shorter/longer.
The **Below Basic** student does not meet the Basic performance level.

### Classify objects and count the number of objects in each category.

**K.MD.G.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.)**

In addition to Proficient, the **Advanced** student is able to construct categories from the given objects and justify reasoning for each category.
The **Proficient** student is able to classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.)
The **Basic** student is able to identify objects from given categories; count the numbers of objects in each category. (Limit category counts to be less than or equal to 10.)
The **Below Basic** student does not meet the Basic performance level.

**K.MD.G.4 Identify U.S. coins by name (pennies, nickels, dimes, and quarters).**

In addition to Proficient, the **Advanced** student is able to identify the value of at least two U.S. coins (pennies, nickels, dimes, and quarters).
The **Proficient** student is able to identify U.S. coins by name (pennies, nickels, dimes, and quarters).
The **Basic** student is able to identify at least two U.S. coins by name (pennies, nickels, dimes, and quarters).
The **Below Basic** student does not meet the Basic performance level.

## GEOMETRY

### Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

**K.G.H.1 Describe objects in the environment using the names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.**

In addition to Proficient, the **Advanced** student is able to using the names of shapes describe the position of one object relative to two or more objects.
The **Proficient** student is able to describe objects in the environment using the names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
The **Basic** student is able to describe the relative positions of objects using terms such as above, below, beside, in front of, behind, and next to.
The **Below Basic** student does not meet the Basic performance level.

**K.G.H.2 Correctly name shapes regardless of their orientations or overall size.**

In addition to Proficient, the **Advanced** student is able to correctly identify shapes within a compound figure.
The **Proficient** student is able to correctly name shapes regardless of their orientations or overall size.

The **Basic** student is able to correctly name shapes: squares, circles, triangles, rectangles, cones, and cubes.
**Assessment Boundary:** does not include hexagon, cylinder, or sphere.
The **Below Basic** student does not meet the Basic performance level.

**K.G.H.3 Identify shapes as two-dimensional or three-dimensional.**

In addition to Proficient, the **Advanced** student is able to justify the difference between two-dimensional and three-dimensional shapes.
The **Proficient** student is able to identify shapes as two-dimensional or three-dimensional.
The **Basic** student is able to identify shapes as two-dimensional.
The **Below Basic** student does not meet the Basic performance level.

## Analyze, compare, create, and compose shapes.

**K.G.I.4 Analyze and compare two- and three-dimensional shapes, using informal language to describe their similarities, differences, and attributes.**

In addition to Proficient, the **Advanced** student is able to group shapes based on attributes and justify reasoning.
The **Proficient** student is able to analyze and compare two- and three-dimensional shapes, using informal language to describe their similarities, differences, and attributes.
The **Basic** student is able to use informal language to describe the similarities, differences, and attributes of corresponding two- or three-dimensional shapes (comparing two-dimensional to two-dimensional and three-dimensional to three-dimensional).
The **Below Basic** student does not meet the Basic performance level.

**K.G.I.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.**

In addition to Proficient, the **Advanced** student is able to predict and test what components are needed to change a two-dimensional shape to a three-dimensional shape (e.g., square to cube).
The **Proficient** student is able to model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
The **Basic** student is able to model two-dimensional shapes in the world by building shapes from components (e.g., sticks and clay balls) or drawing two-dimensional shapes.
The **Below Basic** student does not meet the Basic performance level.

**K.G.I.6 Use simple shapes to compose squares, rectangles, and hexagons.**

In addition to Proficient, the **Advanced** student is able to decompose a two-dimensional figure to determine the shapes that were used to build it.
The **Proficient** student is able to use simple shapes to compose squares, rectangles, and hexagons.
The **Basic** student is able to use given simple shapes to compose squares and rectangles.
The **Below Basic** student does not meet the Basic performance level.

**Companion document to the 2018 Mathematics Content Standards**

# Grade 1 Mathematics Content Standards & PLDs

## GRADE 1 MATH PRACTICES

### MP1 Make sense of problems and persevere in solving them.

**1.MP.1** In first grade, students realize that doing mathematics involve solving problems and discussing how they solved them. Students explain the meaning of a problem and look for ways to solve it. Students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by revisiting their work and asking themselves, "Does this make sense?" or, "Should I try another strategy? Students are also working on increasing stamina as they work on problems.

### MP2 Reason abstractly and quantitatively.

**1.MP.2** Students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning means being able to explain through manipulatives or drawings what a problem means while attending to the meanings of the quantities. Students make meaning of a problem situation and translate into a number sentence.

### MP3 Construct viable arguments and critique the reasoning of others.

**1.MP.3** First graders construct arguments using concrete illustrations referents, such as objects, pictures, drawings, and actions. They also practice their mathematical communication skills as they participate in mathematical discussions involving questions like, "How did you get that?" Explain your thinking, "Why is that true?" They not only explain their own thinking, but listen to others' explanations. They decide if the explanations make sense and ask questions for clarity.

### MP4 Model with mathematics.

**1.MP.4** Students experiment with representing problem situations in multiple ways including using objects, acting out, drawing pictures, numbers, words (mathematical language), making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.

### MP5 Use appropriate tools strategically.

**1.MP.5** Students begin to consider the different tools available when thinking about the concepts of number. They evaluate the available tools (including concrete manipulatives, drawings, estimation, and applications) when solving a mathematical problem and decide when certain tools might be helpful and give a reason for using the tool to solve the problem. For instance, first graders decide it might be best to use colored chips to model an addition problem.

### MP6 Attend to precision.

**1.MP.6** Students begin to develop their mathematical communication skills. They try to use clear and precise mathematical vocabulary in their discussions with others and in their own reasoning. Students learn to express their work with mathematical language and symbols.

### MP7 Look for and make use of structure.

**1.MP.7** First graders begin to discern a number pattern or structure. For instance, if students recognize $12 + 3 = 15$, then they also know $3 + 12 = 15$. (Commutative Property of addition.) To add $4 + 6 + 4$, the first two numbers can be added to make a ten, so $4 + 6 + 4 = 10 + 4 = 14$. Students continue to develop their understanding of patterns in our number system.

### MP8 Look for and express regularity in repeated reasoning.

**1.MP.8** Students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract ten, including multiples of ten, then they notice the pattern and gain a better understanding of place value. Students also notice that when adding two numbers, order of adding doesn't affect the sum (Commutative Property). They also notice that three numbers create a family when adding or subtracting ($2 + 3 = 5$ and $5 - 2 = 3$).

## OPERATIONS AND ALGEBRAIC THINKING

### Represent and solve problems involving addition and subtraction.

**1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, by using objects, drawings, or equations with a symbol for the unknown number to represent the problem.**

In addition to Proficient, the **Advanced** student is able to create and solve an addition or subtraction word problem within 20.
The **Proficient** student is able to use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, by using objects, drawings, or equations with a symbol for the unknown number to represent the problem.
The **Basic** student is able to use addition and subtraction within 10 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, by using objects, drawings, or equations with a symbol for the unknown number to represent the problem.
The **Below Basic** student does not meet the Basic performance level.

**1.OA.A.2 Solve word problems that call for the addition of three whole numbers whose sum is less than or equal to 20, by using objects, drawings, or equations.**

In addition to Proficient, the **Advanced** student is able to solve a missing addend word problem that calls for the addition of three whole numbers whose sum is less than or equal to 20, by using objects, drawings, or equations.
The **Proficient** student is able to solve word problems that call for the addition of three whole numbers whose sum is less than or equal to 20, by using objects, drawings, or equations.
The **Basic** student is able to add three whole numbers whose sum is less than or equal to 20, by using objects, drawings, or equations.
The **Below Basic** student does not meet the Basic performance level.

### Understand and apply properties of operations and the relationship between addition and subtraction.

**1.OA.B.3 Apply Commutative and Associative Properties of addition as strategies to add and subtract.**

In addition to Proficient, the **Advanced** student is able to describe the relationships when applying the properties of addition.
The **Proficient** student is able to apply Commutative and Associative properties of addition as strategies to add and subtract.
The **Basic** student is able to apply Commutative Property of addition.

The **Below Basic** student does not meet the Basic performance level.

**1.OA.B.4 Understand subtraction as an unknown-addend problem.**

In addition to Proficient, the **Advanced** student is able to write all equivalent addition or subtraction equations relating three whole numbers within 20.
The **Proficient** student is able to understand subtraction as an unknown-addend problem.
The **Basic** student is able to understand subtraction as an unknown-addend problem within 10.
The **Below Basic** student does not meet the Basic performance level.

## Add and subtract within 20.

**1.OA.C.5 Relate counting to addition and subtraction using strategies, such as, by counting on and back.**

In addition to Proficient, the **Advanced** student is able to, given a counting on or counting back situation, write related addition or subtraction equations (e.g., $42, 41, 40, 39$. $42 - 3 = 39$, or $42 - 39 = 3$).
The **Proficient** student is able to relate counting to addition and subtraction using strategies, such as, by counting on and back.
The **Basic** student is able to relate counting to addition by counting on.
The **Below Basic** student does not meet the Basic performance level.

**1.OA.C.6 Add and subtract within 20, demonstrating fluency in addition and subtraction within 10. Use strategies such as counting on; making ten using the relationship between addition and subtraction.**

In addition to Proficient, the **Advanced** student is able to add and subtract within 20, demonstrating fluency in addition and subtraction within 10 by utilizing multiple strategies, such as: counting on; making ten using the relationship between addition and subtraction.
The **Proficient** student is able to add and subtract within 20, demonstrating fluency in addition and subtraction within 10. Use strategies such as counting on; making ten using the relationship between addition and subtraction.
The **Basic** student is able to add and subtract within 10 using strategies such as counting on; making ten using the relationship between addition and subtraction.
The **Below Basic** student does not meet the Basic performance level.

## Work with addition and subtraction equations.

**1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.**

In addition to Proficient, the **Advanced** student is able to determine if equations involving addition and subtraction are true or false and rewrite false equations to make them true.
The **Proficient** student is able to understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.
The **Basic** student is able to use the equal sign to make a true addition or subtraction equation given a visual representation of a situation.
The **Below Basic** student does not meet the Basic performance level.

**1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.**

In addition to Proficient, the **Advanced** student is able to describe multiple strategies for determining the unknown whole number in an addition or subtraction equation relating three whole numbers.
The **Proficient** student is able to determine the unknown whole number in an addition or subtraction equation relating three whole numbers.
The **Basic** student is able to determine the unknown whole number in an addition or subtraction equation relating three whole numbers less than 10.
The **Below Basic** student does not meet the Basic performance level.

## NUMBER AND OPERATIONS IN BASE TEN

### Extend the counting sequence.

**1.NBT.E.1 Extend the number sequences to 120. In this range:**

**1.NBT.E.1A** Count forward and backward, starting at any number less than 120.

**1.NBT.E.1B** Read numerals.

**1.NBT.E.1C** Write numerals.

**1.NBT.E.1D** Represent a number of objects with a written numeral.

In addition to Proficient, the **Advanced** student is able to:

- Count forward and backward by 10, starting at any number less than 120. **OR**
- Count forward and backward by 2, starting at any even number less than 120. **OR**
- Count forward and backward by 5, starting at any multiple of 5 less than 120.

The **Proficient** student is able to extend the number sequences to 120. In this range:

A. Count forward and backward, starting at any number less than 120.
B. Read numerals.
C. Write numerals.
D. Represent a number of objects with a written numeral.

The **Basic** student is able to extend the number sequences to 120 with guidance. In this range:

A. Count forward starting at any number less than 120.
B. Read numerals.
C. Write numerals.
D. Represent a number of objects with a written numeral.

The **Below Basic** student does not meet the Basic performance level.

### Understand place value.

**1.NBT.F.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:**

**1.NBT.F.2A** 10 can be thought of as a bundle of ten ones — called a “ten.”

**1.NBT.F.2B** The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

**1.NBT.F.2C** The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

In addition to Proficient, the **Advanced** student is able to represent any two digit number in multiple ways using tens and ones (e.g., 67 is 6 tens and 7 ones, or 4 tens and 27 ones, etc.).

The **Proficient** student is able to understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

A. 10 can be thought of as a bundle of ten ones -- called a “ten.”
B. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
C. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

The **Basic** student is able to build any two digit number using manipulatives to represent amounts of tens and ones and show understanding of the following special cases:

A. 10 can be thought of as a bundle of ten ones -- called a “ten.”
B. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

The **Below Basic** student does not meet the Basic performance level.

**1.NBT.F.3 Compare pairs of two-digit numbers based on the values of the tens digit and the ones digits, recording the results of comparisons with the words "is greater than," "is equal to," "is less than," and with the symbols >, =, and <.**

In addition to Proficient, the **Advanced** student is able to write comparisons recording the results using both the greater than and the less than words and symbols (e.g., $27 > 21$, and $21 < 27$).

The **Proficient** student is able to compare pairs of two-digit numbers based on the values of the tens digit and the ones digit, recording the results of comparisons with the words "is greater than," "is equal to," "is less than," and with the symbols >, =, and <.

The **Basic** student is able to compare pairs of two-digit numbers based on the values of the tens digit and the ones digit, stating the results of comparisons with the words "is greater than," "is equal to," "is less than."

The **Below Basic** student does not meet the Basic performance level.

## Use place value understanding and properties of operations to add and subtract.

**1.NBT.G.4 Add within 100, using concrete models or drawings and strategies based on place value:**

**1.NBT.G.4A** Including adding a two-digit number and a one-digit number.
**1.NBT.G.4B** Adding a two-digit number and a multiple of 10.
**1.NBT.G.4C**Understand that in adding two-digit numbers, adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
**1.NBT.G.4D** Relate the strategy to a written method and explain the reasoning used.

In addition to Proficient, the **Advanced** student is able to subtract within 100, using concrete models or drawings and strategies based on place value:

A. Including subtracting a one-digit number from a two-digit number.
B. Subtracting a multiple of 10 from a two-digit number.

The **Proficient** student is able to add within 100, using concrete models or drawings and strategies based on place value:

A. Including adding a two-digit number and a one-digit number.
B. Adding a two-digit number and a multiple of 10.
C. Understand that in adding two-digit numbers, adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
D. Relate the strategy to a written method and explain the reasoning used.

The **Basic** student is able to add within 100, using concrete models, manipulatives, or drawings and strategies based on place value:

A. Including adding a two-digit number and a one-digit number.
B. Adding a two-digit number and a multiple of 10.

The **Below Basic** student does not meet the Basic performance level.

**1.NBT.G.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.**

In addition to Proficient, the **Advanced** student is able to given a two-digit number, mentally find multiples of 10 more or multiples of 10 less than the number, without having to count; explain the reasoning used.
The **Proficient** student is able to given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
The **Basic** student is able to mentally find 10 more or 10 less than a multiple of 10 less than 100, without having to count; explain the reasoning used.
The **Below Basic** student does not meet the Basic performance level.

**1.NBT.G.6 Subtract multiples of 10 from an equal or larger multiple of 10 both in the range 10-90, using concrete models, drawings, and strategies based on place value.**

In addition to Proficient, the **Advanced** student is able to, without counting, subtract multiples of ten and explain the reasoning used.
The **Proficient** student is able to subtract multiples of 10 from an equal or larger multiple of 10 both in the range 10-90, using concrete models, drawings, and strategies based on place value.
The **Basic** student is able to, starting at any multiple of ten in the range 10-90, verbally count backwards by tens.
The **Below Basic** student does not meet the Basic performance level.

## MEASUREMENT AND DATA

### Measure lengths indirectly and by iterating length units.

**1.MD.H.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.**

In addition to Proficient, the **Advanced** student is able to, given different organizations of more than three objects, justify an arrangement of the objects by length.
The **Proficient** student is able to order three objects by length; compare the lengths of two objects indirectly by using a third object.
The **Basic** student is able to compare two objects by length.
The **Below Basic** student does not meet the Basic performance level.

**1.MD.H.2 Use nonstandard units to show the length of an object as the number of same size units of length with no gaps or overlaps.**

In addition to Proficient, the **Advanced** student is able to measure the same object using a non-standard unit of one length and then a non-standard unit of a different length. Explain how the two measurements relate to the size of the unit chosen.
The **Proficient** student is able to use nonstandard units to show the length of an object as the number of same size units of length with no gaps or overlaps.
The **Basic** student is able to use nonstandard units to show the length of an object as the number of same size units of length with no gaps or overlaps with guidance.
The **Below Basic** student does not meet the Basic performance level.

### Work with time and money.

**1.MD.I.3A Tell and write time in hours and half-hours using analog and digital clocks.**

In addition to Proficient, the **Advanced** student is able to approximate time to the nearest hour or half hour on an analog clock based only on the hour hand.
The **Proficient** student is able to tell and write time in hours and half-hours using analog and digital clocks.
The **Basic** student is able to tell and write time in hours using analog and digital clocks.
The **Below Basic** student does not meet the Basic performance level.

**1.MD.I.3B Identify U.S. coins by value (pennies, nickels, dimes, quarters).**

In addition to Proficient, the **Advanced** student is able to find equivalent values of coins up to and including quarters using coins of lesser value.
The **Proficient** student is able to identify U.S. coins by value (pennies, nickels, dimes, quarters).
The **Basic** student is able to identify at least two U.S. coins by value (pennies, nickels, dimes, quarters).
The **Below Basic** student does not meet the Basic performance level.

### Represent and interpret data.

**1.MD.J.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.**

In addition to Proficient, the **Advanced** student is able to compare two different data sets with at least three categories to ask and answer questions.
The **Proficient** student is able to organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
The **Basic** student is able to ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another when given an organized set of data.
The **Below Basic** student does not meet the Basic performance level.

## GEOMETRY

### Reason with shapes and their attributes.

**1.G.K.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); for a wide variety of shapes; build and draw shapes to possess defining attributes.**

In addition to Proficient, the **Advanced** student is able to compare and contrast defining attributes from given shapes and use the comparison to change one shape into the other shape.
The **Proficient** student is able to distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); for a wide variety of shapes; build and draw shapes to possess defining attributes.
The **Basic** student is able to identify attributes (both defining and non-defining) for a wide variety of shapes.
The **Below Basic** student does not meet the Basic performance level.

**1.G.K.2 Use two-dimensional shapes (rectangles, squares, trapezoids, rhombuses, and triangles) or three-dimensional shapes (cubes, rectangular prisms, cones, and cylinders) to create a composite figure, and create new figures from the composite figure.**

In addition to Proficient, the **Advanced** student is able to decompose a composite figure (made of two and three dimensional shapes) and then compose to the original figure and create a new composite figure.

The **Proficient** student is able to use two-dimensional shapes (rectangles, squares, trapezoids, rhombuses, and triangles) or three-dimensional shapes (cubes, rectangular prisms, cones, and cylinders) to create a composite figure, and create new figures from the composite figure.

The **Basic** student is able to use two-dimensional shapes (rectangles, squares, trapezoids, rhombuses, and triangles) to create a composite figure.

The **Below Basic** student does not meet the Basic performance level.

**1.G.K.3 Partition circles and rectangles into two and four equal shares and:**

**1.G.K.3A** Describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of.

**1.G.K.3B** Describe the whole as two of, or four of the shares.

**1.G.K.3C** Recognize that decomposing into more equal shares creates smaller shares.

In addition to Proficient, the **Advanced** student is able to partition multiple circles and rectangles into two and four equal shares and:

A. Describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of.
B. Describe the whole as two of, or four of the shares.
C. Recognize that decomposing into more equal shares creates smaller shares.

The **Proficient** student is able to partition circles and rectangles into two and four equal shares and:

A. Describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of.
B. Describe the whole as two of, or four of the shares.
C. Recognize that decomposing into more equal shares creates smaller shares.

The **Basic** student is able to partition circles and rectangles into two equal shares and:

A. Describe the shares using the word halves.
B. Describe the whole as two of the shares.

The **Below Basic** student does not meet the Basic performance level.

**Companion document to the 2018 Mathematics Content Standards**

# Grade 2 Mathematics Content Standards & PLDs

## GRADE 2 MATH PRACTICES

### MP1 Make sense of problems and persevere in solving them.

**2.MP.1** In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They make conjectures about the solution and plan out a problem-solving approach. Students work on increasing stamina.

### MP2 Reason abstractly and quantitatively.

**2.MP.2** Students recognize that a number represents a specific quantity and connect the quantity to written symbols. Quantitative reasoning entails being able to explain through manipulatives or drawings what a problem means, while attending to the meanings of the quantities. Students make meaning of a problem situation and translate into a number sentence. Second graders begin to know and use different properties of operations and relate addition and subtraction.

### MP3 Construct viable arguments and critique the reasoning of others.

**2.MP.3** Second graders may construct arguments using concrete illustrations, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like, "How did you get that?" Explain your thinking, "Why is that true?" They not only explain their own thinking, but listen to others' explanations and compare strategies. They decide if the explanations make sense and ask appropriate questions for clarity.

### MP4 Model with mathematics.

**2.MP.4** Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.

### MP5 Use appropriate tools strategically.

**2.MP.5** Students decide how and when to use the available tools appropriately and efficiently when solving a mathematical problem. Students reason whether or not a tool was helpful in solving the problem. For instance, second graders may decide to solve a problem by drawing a picture rather than writing an equation.

### MP6 Attend to precision.

**2.MP.6** Students begin to develop their mathematical communication skills, (orally and written) They use clear and precise mathematical language and symbols when explaining their own reasoning.

### MP7 Look for and make use of structure.

**2.MP.7** Second graders look for patterns. For instance, they adopt mental math strategies based on patterns (making ten, fact families, doubles, adding and subtracting numbers by place, and equal shares). Their understanding of the number system develops into 3- and 4- digit numbers.

### MP8 Look for and express regularity in repeated reasoning.

**2.MP.8** Second grade students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract, they look for shortcuts, such as tens are added to tens, ones are added to ones, and sometimes the ones make a new ten. They also notice when a whole is shared into equal groups, the size of the share gets smaller the more shares.

## OPERATIONS AND ALGEBRAIC THINKING

### Represent and solve problems involving addition and subtraction.

**2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, by using drawings and equations with a symbol for the unknown number to represent the problem.**

In addition to Proficient, the **Advanced** student is able to express solutions to one- and two-step word problems using multiple representations (e.g., $32 + g = 50$ and $g = 50 - 32$).

The **Proficient** student is able to use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, and taking apart, and comparing with unknowns in all positions, by using drawings and equations with a symbol for the unknown number to represent the problem.

The **Basic** student is able to use addition within 100 to solve one- and two-step word problems involving situations of adding to and putting together, by using drawings and equations with a symbol for the unknown number to represent the problem.

The **Below Basic** student does not meet the Basic performance level.

### Add and subtract within 20.

**2.OA.B.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know automatically all sums of two one-digit numbers based on strategies.**

In addition to Proficient, the **Advanced** student is able to fluently add and subtract within 100 using mental strategies.

The **Proficient** student is able to fluently add and subtract within 20 using mental strategies. By end of Grade 2, know automatically all sums of two one-digit numbers based on strategies.

The **Basic** student is able to fluently add and subtract within 20 using mental strategies.

The **Below Basic** student does not meet the Basic performance level.

### Work with equal groups of objects to gain foundations for multiplication.

**2.OA.C.3 Determine whether a group (up to 20) has an odd or even number of objects (i.e. by pairing objects or counting them by 2s).**

**2.OA.C.3A** If the number of objects is even, then write an equation to express this as the sum of two equal addends.

**2.OA.C.3B** If the number of objects group is odd, then write an equation to express this as a sum of a near double (double plus 1).

In addition to Proficient, the **Advanced** student is able to determine whether a group (up to 100) has an odd or even number of objects.

A. If the number of objects is even, then write an equation to express this as the sum of two equal addends.
B. If the number of objects in a group is odd, then write an equation to express this as a sum of a near double (double plus 1).

The **Proficient** student is able to determine whether a group (up to 20) has an odd or even number of objects (i.e. by pairing objects or counting them by 2s).

A. If the number of objects is even, then write an equation to express this as the sum of two equal addends.
B. If the number of objects in a group is odd, then write an equation to express this as a sum of a near double (double plus 1).

The **Basic** student is able to determine whether a group (up to 20) has an odd or even number of objects (i.e. by pairing objects or counting them by 2s).

The **Below Basic** student does not meet the Basic performance level.

**2.OA.C.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.**

In addition to Proficient, the **Advanced** student is able to write at least two equations to express the total as the sum of equal addends (e.g., a $5 + 5 + 5 + 5$ array is the same as $10 + 10$ array).

The **Proficient** student is able to use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

The **Basic** student is able to use addition to find the total number of objects arranged in rectangular arrays with no more than 10 objects; write an equation to express the total as a sum of equal addends.

The **Below Basic** student does not meet the Basic performance level.

## NUMBER AND OPERATIONS IN BASE TEN

### Understand place value.

**2.NBT.D.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; and demonstrate that cases: a. 100 can be thought of as a bundle of ten tens — called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).**

**2.NBT.D.1A** 100 can be thought of as a bundle of ten tens — called a "hundred."

**2.NBT.D.1B** The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

**2.NBT.D.1C** Three-digit numbers can be decomposed in multiple ways (e.g., 524 can be decomposed as 5 hundreds, 2 tens and 4 ones or 4 hundreds, 12 tens, and 4 ones, etc.)

In addition to Proficient, the **Advanced** student is able to understand that the four digits of a four-digit number represent amounts of thousands, hundreds, tens, and ones; and demonstrate that:

A. 1000 can be thought of as a bundle of ten hundreds -- called a "thousand."

B. Multiples of one hundred larger than 1000 can be referred to both as a count of hundreds or by place value with thousands and hundreds (e.g., 4200 is forty-two hundred and four thousand, two hundred).

C. Four-digit numbers can be decomposed in multiple ways (e.g., 2524 can be decomposed as 2 thousands, 5 hundreds, 2 tens and 4 ones or 25 hundreds, 2 tens, and 4 ones, etc.)

The **Proficient** student is able to understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; and demonstrate that:

A. 100 can be thought of as a bundle of ten tens — called a "hundred."

B. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

C. Three-digit numbers can be decomposed in multiple ways (e.g., 524 can be decomposed as 5 hundreds, 2 tens and 4 ones or 4 hundreds, 12 tens, and 4 ones, etc.)

The **Basic** student is able to understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; and demonstrate that:

A. 100 can be thought of as a bundle of ten tens — called a "hundred."

B. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

C. Three-digit numbers can be decomposed into place value (e.g., 524 can be decomposed as 5 hundreds, 2 tens, and 4 ones, etc.)

The **Below Basic** student does not meet the Basic performance level.

**2.NBT.D.2 Skip-count by 10s and 100s within 1000 starting at any given number.**

In addition to Proficient, the **Advanced** student is able to skip-count by multiple units of 10s and 100s within1000 starting at any given number.
The **Proficient** student is able to skip-count by 10s and 100s within 1000 starting at any given number.
The **Basic** student is able to skip-count by 10s and 100s within 1000 starting at any multiple of 10 or multiple of 100.
The **Below Basic** student does not meet the Basic performance level.

**2.NBT.D.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.**

In addition to Proficient, the **Advanced** student is able to read and write numbers to 10000 using base-ten numerals, number names, and expanded form. **OR** Represent a three digit number in multiple ways using base 10 numerals or expanded form (e.g., 674 could be: $600 + 70 + 4$ or $500 + 170 + 4$ or $600 + 60 + 14$).
The **Proficient** student is able to read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
The **Basic** student is able to read and write numbers to 1000 using base-ten numerals and number names and write numbers to 100 using expanded form.
The **Below Basic** student does not meet the Basic performance level.

**2.NBT.D.4 Compare pairs of three-digit numbers based on meanings of the hundreds, tens, and ones digits, using the words "is greater than," "is equal to," "is less than," and with the symbols $>$, $=$, and $<$ to record the results of comparisons.**

In addition to Proficient, the **Advanced** student is able to write comparisons recording the results using both the greater than and the less than words and symbols (e.g., $127 > 121$, and $121 < 127$).
The **Proficient** student is able to compare pairs of three-digit numbers based on meanings of the hundreds, tens, and ones digits, using the words "is greater than," "is equal to," "is less than" and with the symbols $>$, $=$, and $<$ to record the results of comparisons.
The **Basic** student is able to compare pairs of three-digit numbers from a pictorial representation based on meanings of the hundreds, tens, and ones digits, using the words "is greater than," "is equal to," "is less than" and with the symbols $>$, $=$, and $<$ to record the results of comparisons.
The **Below Basic** student does not meet the Basic performance level.

## Use place value understanding and properties of operations to add and subtract.

**2.NBT.E.5 Add and subtract within 100 using strategies based on place value, properties of addition, and/or the relationship between addition and subtraction.**

In addition to Proficient, the **Advanced** student is able to add and subtract within 100 using **multiple** strategies based on place value, properties of addition, and the relationship between addition and subtraction.
The **Proficient** student is able to add and subtract within 100 using strategies based on place value, properties of addition, and/or the relationship between addition and subtraction.
The **Basic** student is able to add and subtract within 100 in problems that do not require regrouping using strategies based on place value, properties of addition, and/or the relationship between addition and subtraction.
The **Below Basic** student does not meet the Basic performance level.

**2.NBT.E.6 Add up to four two-digit numbers using strategies based on place value and/or properties of addition.**

In addition to Proficient, the **Advanced** student is able to add up at least four two-digit numbers using **multiple** strategies based on place value and properties of addition.

The **Proficient** student is able to add up to four two-digit numbers using strategies based on place value and/or properties of addition.

The **Basic** student is able to add up to four one-digit numbers with sums greater than 20 using strategies based on place value and/or properties of addition.

The **Below Basic** student does not meet the Basic performance level.

**2.NBT.E.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of addition, and/or the relationship between addition and subtraction:**

**2.NBT.E.7A** Relate the strategy to a written method and explain the reasoning used.

**2.NBT.E.7B** Understand that in adding or subtracting three-digit numbers, add or subtract hundreds and hundreds, tens and tens, ones and ones.

**2.NBT.E.7C** Understand that sometimes it is necessary to compose or decompose tens or hundreds.

In addition to Proficient, the **Advanced** student is able to add and subtract within 1000 using **multiple** strategies based on place value, properties of addition, and the relationship between addition and subtraction. Relate the strategies used to written methods and explain the reasoning.

The **Proficient** student is able to add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of addition, and/or the relationship between addition and subtraction:

A. Relate the strategy to a written method and explain the reasoning used.
B. Understand that in adding or subtracting three-digit numbers, add or subtract hundreds and hundreds, tens and tens, ones and ones.
C. Understand that sometimes it is necessary to compose or decompose tens or hundreds.

The **Basic** student is able to add and subtract within 1000 in problems that do not require regrouping, using concrete models or drawings and strategies based on place value, properties of addition, and/or the relationship between addition and subtraction.

The **Below Basic** student does not meet the Basic performance level.

**2.NBT.E.8**

**2.NBT.E.8A Mentally, add 10 or 100 to a given number 100-900.**

**2.NBT.E.8B Mentally, subtract 10 or 100 from a given number 100-900.**

In addition to Proficient, the **Advanced** student is able to, mentally:

A. Add multiple units of 10s or 100s to a given number 100-900, and
B. Subtract multiple units of 10s or 100s from a given number 100-900.

The **Proficient** student is able to, mentally:

A. Add 10 or 100 to a given number 100-900, and
B. Subtract 10 or 100 from a given number 100-900.

The **Basic** student is able to, mentally:

A. Add 10 or 100 to a given multiple of 10 or multiple of 100 in the range 100-900, and
B. Subtract 10 or 100 from a given multiple of 10 or multiple of 100 in the range 100-900.

The **Below Basic** student does not meet the Basic performance level.

**2.NBT.E.9 Explain why addition and subtraction strategies work, using place value and the properties of addition. (Explanations may be supported by drawings, objects, or written form.)**

In addition to Proficient, the **Advanced** student is able to evaluate different addition and subtraction strategies for a given situation, determine the most efficient strategy, and justify reasoning.
The **Proficient** student is able to explain why addition and subtraction strategies work, using place value and the properties of addition. (Explanations may be supported by drawings, objects, or written form.)
The **Basic** student is able to explain why addition strategies work, using place value and the properties of addition. (Explanations may be supported by drawings, objects, or written form.)
The **Below Basic** student does not meet the Basic performance level.

## MEASUREMENT AND DATA

### Measure and estimate lengths in standard units.

**2.MD.F.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.**

In addition to Proficient, the **Advanced** student is able to use the most appropriate measurement tool and provide justification for the selection.
The **Proficient** student is able to measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
The **Basic** student is able to measure the length of an object with a given tool.
The **Below Basic** student does not meet the Basic performance level.

**2.MD.F.2 Measure the same object or distance using a standard unit of one length and then a standard unit of a different length. Explain how the two measurements relate to the size of the unit chosen.**

In addition to Proficient, the **Advanced** student is able to measure an object or distance using a standard unit of length and describe other measurements that could be used to measure the length in another unit of the same system. Explain the relationship of the units. Students are not required to know direct unit conversions.
The **Proficient** student is able to measure the same object or distance using a standard unit of one length and then a standard unit of a different length. Explain how the two measurements relate to the size of the unit chosen.
The **Basic** student is able to measure an object or distance using a standard unit.
The **Below Basic** student does not meet the Basic performance level.

**2.MD.F.3 Estimate lengths using units of inches, feet, centimeters, and meters.**

In addition to Proficient, the **Advanced** student is able to give an estimation and provide justification for the unit selection, when given a real-world scenario.
The **Proficient** student is able to estimate lengths using units of inches, feet, centimeters, and meters.
The **Basic** student is able to estimate lengths using units of inches, feet, centimeters, and meters when given a scaled pictorial comparison of one unit and a given object (e.g., the student is asked to estimate the length of a car in meters, when given a pictorial representation of a meter stick in relation to the car)
The **Below Basic** student does not meet the Basic performance level.

**2.MD.F.4 Measure in standard length units to determine how much longer one object is than another.**

In addition to Proficient, the **Advanced** student is able to measure three or more lengths and order or compare the measurements.
The **Proficient** student is able to measure in standard length units to determine how much longer one object is than another.
The **Basic** student is able to identify longer or shorter given two lengths.
The **Below Basic** student does not meet the Basic performance level.

## Relate addition and subtraction to length.

**2.MD.G.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units.**

In addition to Proficient, the **Advanced** student is able to express multiple representations of solutions to addition and subtraction word problems involving lengths (e.g., $32 + g = 50$ and $g = 50 - 32$).
The **Proficient** student is able to use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units.
The **Basic** student is able to use addition within 100 to solve word problems involving lengths that are given in the same units.
The **Below Basic** student does not meet the Basic performance level.

**2.MD.G.6 Use a number line diagram with equally spaced points to:**

**2.MD.G.6A** Represent whole-number sums and differences within 100 on a number line diagram.
**2.MD.G.6B** Locate the multiple of 10 before and after a given number within 100.

In addition to Proficient, the **Advanced** student is able to use multiple strategies to solve whole-number sums and differences within 100 on a number line diagram.
The **Proficient** student is able to use a number line diagram with equally spaced points to:

A. Represent whole-number sums and differences within 100 on a number line diagram.
B. Locate the multiple of 10 before and after a given number within 100.

The **Basic** student is able to use a number line diagram with equally spaced points to locate the multiple of 10 before and after a given number within 100.
The **Below Basic** student does not meet the Basic performance level.

## Work with time and money.

**2.MD.H.7 Tell and write time from analog and digital clocks in five minute increments using a.m. and p.m.**

In addition to Proficient, the **Advanced** student is able to determine the amount of time until the next hour.
The **Proficient** student is able to tell and write time from analog and digital clocks in five minute increments using a.m. and p.m.
The **Basic** student is able to match corresponding times in five minute increments on analog and digital clocks.
The **Below Basic** student does not meet the Basic performance level.

**2.MD.H.8 Solve word problems up to $10 involving dollar bills, quarters, dimes, nickels, and pennies, using $ (dollars) and ¢ (cents) symbols appropriately.**

In addition to Proficient, the **Advanced** student is able to count up from an amount (using the least amount of bills and coins) to determine the change from a 10 dollar bill.
The **Proficient** student is able to solve word problems up to $10 involving dollar bills, quarters, dimes, nickels, and pennies, using $ (dollars) and ¢ (cents) symbols appropriately.
The **Basic** student is able to solve word problems up to $10 involving dollar bills and dimes using $ (dollars) and ¢ (cents) symbols appropriately.
The **Below Basic** student does not meet the Basic performance level.

## Represent and interpret data.

**2.MD.I.9 Generate measurement data based on whole units and show data by making a line plot.**

In addition to Proficient, the **Advanced** student is able to answer addition and subtraction questions that compare measurements using information presented in a line plot.
The **Proficient** student is able to generate measurement data based on whole units and show data by making a line plot.
The **Basic** student is able to place given measurement data on a line plot.
The **Below Basic** student does not meet the Basic performance level.

**2.MD.I.10**

**2.MD.I.10A Use data to draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories.**
**2.MD.I.10B Use data to solve simple put-together, take-apart, and compare problems using information presented in a bar graph.**

In addition to Proficient, the **Advanced** student is able to solve put-together, take-apart, and compare problems using information presented in multiple bar and/or picture graphs.
The **Proficient** student is able to use data to:

A. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories.
B. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

The **Basic** student is able to use data to:

A. Draw a picture graph or a bar graph (with single-unit scale) to represent a data set with up to four categories.
B. Solve simple put-together problems using information presented in a bar graph.

The **Below Basic** student does not meet the Basic performance level.

## GEOMETRY

### Reason with shapes and their attributes.

**2.G.J.1 Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. (Sizes are compared directly or visually, not compared by measuring.)**

In addition to Proficient, the **Advanced** student is able to compare and contrast sets of defining attributes from given shapes and use the comparison to change one shape into the other shape.
The **Proficient** student is able to identify triangles, quadrilaterals, pentagons, hexagons, and cubes. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. (Sizes are compared directly or visually, not compared by measuring.)
The **Basic** student is able to identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
The **Below Basic** student does not meet the Basic performance level.

**2.G.J.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.**

In addition to Proficient, the **Advanced** student is able to manipulate a given amount of same-size squares (up to 25) into all of the different possible combinations of rectangles and represent the rectangle combinations with repeated addition equations or expressions.
The **Proficient** student is able to partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
The **Basic** student is able to partition a rectangle into rows **or** columns of same-size squares and count to find the total number of them.
The **Below Basic** student does not meet the Basic performance level.

**2.G.J.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.**

**2.G.J.3A** Describing the shares using the words halves, thirds, half of, a third of, etc.
**2.G.J.3B** Describing the whole as two halves, three thirds, four fourths.
**2.G.J.3C** Recognizing that equal shares of identical wholes need not have the same shape.

In addition to Proficient, the **Advanced** student is able to partition symmetrical shapes into two, three, or four equal shares and describe shares using correct fractional words **or** partition multiple circles and rectangles into two, three, and four equal shares and:

A. Describing the shares using the words halves, thirds, half of, a third of, etc.
B. Describing the whole as two halves, three thirds, four fourths.
C. Recognizing that equal shares of identical wholes need not have the same shape.

The **Proficient** student is able to partition circles and rectangles into two, three, or four equal shares by:

A. Describing the shares using the words halves, thirds, half of, a third of, etc.
B. Describing the whole as two halves, three thirds, four fourths.
C. Recognizing that equal shares of identical wholes need not have the same shape.

The **Basic** student is able to partition rectangles into two, three, or four equal shares by:

A. Describing the shares using the words halves, thirds, half of, a third of, etc.
B. Describing the whole as two halves, three thirds, four fourths.

The **Below Basic** student does not meet the Basic performance level.

**Companion document to the 2018 Mathematics Content Standards**

# Grade 3 Mathematics Content Standards & PLDs

## GRADE 3 MATH PRACTICES

### MP1 Make sense of problems and persevere in solving them.

**3.MP.1** In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third grade students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense? "They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

### MP2 Reason abstractly and quantitatively.

**3.MP.2** Students recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.

### MP3 Construct viable arguments and critique the reasoning of others.

**3.MP.3** Students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions such as, "How did you get that? and "Why is that true?" They explain their thinking to others and respond to others' thinking.

### MP4 Model with mathematics.

**3.MP.4** Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Students should evaluate their results in the context of the situation and reflect on whether the results make sense.

### MP5 Use appropriate tools strategically.

**3.MP.5** Students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.

### MP6 Attend to precision.

**3.MP.6** As students develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.

### MP7 Look for and make use of structure.

**3.MP.7** Students look closely to discover a pattern or structure. For example, students use properties of operations as strategies to multiply and divide (Commutative and Distributive Properties).

### MP8 Look for and express regularity in repeated reasoning.

**3.MP.8** Students notice repetitive actions in computation and look for shortcut methods. For example, students may use the Distributive Property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of $7 \cdot 8$, they might decompose $7$ into $5$ and $2$ then multiply $5 \cdot 8$ and $2 \cdot 8$ to arrive at $40 + 16$ or $56$. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

## OPERATIONS AND ALGEBRAIC THINKING

### Represent and solve problems involving multiplication and division.

**3.OA.A.1 Represent the concept of multiplication of whole numbers using models including, but not limited to, equal-sized groups ("groups of"), arrays, area models, repeated addition, and equal "jumps" on a number line.**

In addition to Proficient, the **Advanced** student is able to:

- Represent the concept of multiplication of whole numbers using models and strategies in multiple ways.
- Create or write a scenario or model that represents the concept of multiplication of whole numbers.

The **Proficient** student is able to represent the concept of multiplication of whole numbers using models and strategies.
The **Basic** student is able to represent the concept of multiplication of whole numbers using models and strategies with partial success.
The **Below Basic** student may be able to recognize that a model represents the concept of multiplication of a whole number but may require counting individual items or other methods to solve the problem.

**3.OA.A.2 Represent the concept of division of whole numbers (resulting in whole number quotients) using models including, but not limited to, partitioning, repeated subtraction, sharing, and inverse of multiplication.**

In addition to Proficient, the **Advanced** student is able to:

- Represent the concept of division of whole numbers (resulting in whole number quotients) using models and strategies in multiple ways.
- Create or write a scenario or model that represents the concept of division of whole numbers (resulting in whole number quotients).

The **Proficient** student is able to represent the concept of division of whole numbers (resulting in whole number quotients) using models and strategies.
The **Basic** student is able to represent the concept of division of whole numbers (resulting in whole number quotients) using models and strategies with partial success.
The **Below Basic** student may be able to recognize that a model represents the concept of division of a whole number.

**3.OA.A.3 Solve multiplication and division word problems within 100 using appropriate modeling strategies and equations.**

In addition to Proficient, the **Advanced** student is able to:

- Solve two-step multiplication and division word problems (with products and dividends) within 100 using appropriate modeling strategies and equations.
- Solve multiplication and division word problems (with products and dividends) beyond 100 using appropriate modeling strategies and equations.
- Write and solve a multiplication or division word problem (with products and dividends) within or beyond 100 using appropriate modeling strategies and equations.

The **Proficient** student is able to solve multiplication and division word problems (with products and dividends) within 100 using appropriate modeling strategies and equations.

The **Basic** student is able to, when provided a pictorial representation, solve multiplication and division word problems (with products and dividends) within 100 using appropriate modeling strategies and equations.

The **Below Basic** student may be able to select the appropriate operation necessary to solve a multiplication or division word problem (with products or dividends) within 100.

**3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is a missing factor, product, dividend, divisor, or quotient. (Students need not know formal terms.)**

In addition to Proficient, the **Advanced** student is able to determine the unknown whole number in a multiplication or division equation relating at least one three digit whole number when the unknown is a missing factor, product, dividend, divisor, or quotient.

The **Proficient** student is able to determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is a missing factor, product, dividend, divisor, or quotient. (Students need not know formal terms.)

The **Basic** student is able to determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is a missing factor, product, dividend, divisor, or quotient with partial success.

The **Below Basic** student may be able to, when provided pictorial support, determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is a missing factor, product, dividend, divisor, or quotient.

## Understand properties of multiplication and the relationship between multiplication and division.

**3.OA.B.5 Apply properties of multiplication as strategies to multiply and divide. (Students need not use formal terms for these properties.)**

In addition to Proficient, the **Advanced** student is able to apply properties of multiplication and division to:

- Multiply two-digit numbers.
- Divide dividends greater than 100 by a one-digit divisor.

The **Proficient** student is able to apply properties of multiplication including Commutative, Associative, and Distributive as strategies to multiply and divide. (Students need not use formal terms for these properties.)

The **Basic** student is able to apply at least one of the properties of multiplication including Commutative, Associative, and Distributive as strategies to multiply and divide.

The **Below Basic** student may be able to recognize the Commutative Property of multiplication.

**3.OA.B.6 Understand division as an unknown-factor problem.**

In addition to Proficient, the **Advanced** student is able to apply the relationship between multiplication and division to find an unknown with a dividend of at least three digits.

The **Proficient** student is able to apply the relationship between multiplication and division to find the unknown.

The **Basic** student is able to apply the relationship between multiplication and division to find an unknown factor of a division problem when provided fact families.

The **Below Basic** student may be able to apply the relationship between multiplication and division to find an unknown factor of a division problem when provided fact families with partial success.

## Multiply and divide within 100.

**3.OA.C.7 Fluently multiply and divide with factors 1 - 10 using mental strategies. By end of Grade 3, know automatically all products of one-digit factors based on strategies.**

In addition to Proficient, the **Advanced** student is able to fluently multiply and divide two numbers with at least one factor greater than 10 using mental strategies.

The **Proficient** student is able to fluently multiply and divide with factors 1 - 10 using mental strategies. By end of Grade 3, know automatically all products of one-digit factors based on strategies.

The **Basic** student is able to multiply and divide with factors 1 - 10 using mental strategies and/or pictorial representation.

The **Below Basic** student may be able to multiply and divide with factors 1 - 10 using mental strategies and/or pictorial representation with partial success.

## Solve problems involving the four operations, and identify and explain patterns in arithmetic.

**3.OA.D.8 Solve two-step word problems (limited to the whole number system) using the four basic operations. Students should apply the Order of Operations when there are no parentheses to specify a particular order.**

**3.OA.D.8A** Represent these problems using equations with a symbol standing for the unknown quantity.

**3.OA.D.8B** Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

In addition to Proficient, the **Advanced** student is able to given an equation with at least two operations, create and solve a word problem that matches the equation.

The **Proficient** student is able to solve two-step word problems (limited to the whole number system) using the four basic operations. Students should apply the Order of Operations when there are no parentheses to specify a particular order.

A. Represent these problems using equations with a symbol standing for the unknown quantity.

B. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

The **Basic** student is able to, when provided a pictorial representation, solve two-step word problems (limited to the whole number system) using the four basic operations. Students should apply the Order of Operations when there are no parentheses to specify a particular order.

The **Below Basic** student may be able to, when provided a pictorial representation, solve two-step word problems (limited to the whole number system) using the four basic operations. Students should apply the Order of Operations when there are no parentheses to specify a particular order with partial success.

**3.OA.D.9 Identify arithmetic patterns and explain the relationships using properties of operations.**

In addition to Proficient, the **Advanced** student is able to:

- Identify a characteristic of a pattern that is not explicitly given. **OR**
- Write an equation to find the nth term of the arithmetic pattern. **OR**
- Identify arithmetic patterns and explain the relationships using properties of operations from a real-world problem.

The **Proficient** student is able to identify arithmetic patterns and explain the relationships using properties of operations.
The **Basic** student is able to predict the next term of a pattern.
The **Below Basic** student may be able to predict the next term of a pattern with partial success.

## NUMBER AND OPERATIONS IN BASE TEN

### Use place value understanding and properties of operations to perform multi-digit arithmetic (a range of algorithms may be used).

**3.NBT.E.1 Use place value understanding to round whole numbers to the nearest 10 or 100.**

In addition to Proficient, the **Advanced** student is able to:

- Round to the nearest 1,000. **OR**
- Round a multi-digit number to a given place value(s).

The **Proficient** student is able to use place value understanding to round whole numbers to the nearest 10 or 100.
The **Basic** student is able to round to the nearest 10 or 100 when provided a model such as a number line.
The **Below Basic** student may be able to round to the nearest 10 or 100 with partial success when provided a model such as a number line.

**3.NBT.E.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of addition, and/or the relationship between addition and subtraction.**

In addition to Proficient, the **Advanced** student is able to fluently add and subtract beyond 1,000 using strategies and algorithms based on place value, properties of addition, and/or the relationship between addition and subtraction.
The **Proficient** student is able to fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of addition, and/or the relationship between addition and subtraction.
The **Basic** student is able to add and subtract within 1000 using strategies and algorithms based on place value, properties of addition, and/or the relationship between addition and subtraction.
The **Below Basic** student may be able to add and subtract within 1000 using strategies and algorithms based on place value, properties of addition, and/or the relationship between addition and subtraction with partial success.

**3.NBT.E.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., $9 \cdot 80, 5 \cdot 60$) using strategies based on place value and properties of multiplication.**

In addition to Proficient, the **Advanced** student is able to multiply 2-digit whole numbers (less than 20) by multiples of 10.
The **Proficient** student is able to multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., $9 \cdot 80, 5 \cdot 60$) using strategies based on place value and properties of multiplication.
The **Basic** student is able to multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., $9 \cdot 80, 5 \cdot 60$) when given a model or strategy based on place value.

The **Below Basic** student may be able to multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., $9 \cdot 80$, $5 \cdot 60$) with partial success when given a model or strategy based on place value.

## NUMBER AND OPERATIONS - FRACTIONS

### Develop understanding of fractions as numbers. (Limited to denominators 2, 3, 4, 6, and 8) *use horizontal fractions.

**3.NF.F.1 Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.**

In addition to Proficient, the **Advanced** student is able to, when given a fraction greater than 1 whole, understand a fraction $\frac{1}{b}$ as a quantity formed by one part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

**Assessment Boundary:** Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

The **Proficient** student is able to understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

**Assessment Boundary:** Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

The **Basic** student is able to identify $\frac{1}{b}$ when given a pictorial representation of a whole partitioned into equal parts.

**Assessment Boundary:** Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

The **Below Basic** student may be able to identify $\frac{1}{b}$ with partial success when given a pictorial representation of a whole partitioned into equal parts.

**Assessment Boundary:** Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

**3.NF.F.2 Understand and represent fractions on a number line diagram.**

**3.NF.F.2A** Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

**3.NF.F.2B** Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

In addition to Proficient, the **Advanced** student is able to understand and represent fractions on a number line diagram when extending to fractions beyond one whole:

A. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to beyond 1 and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

B. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

**Assessment Boundary:** Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

The **Proficient** student is able to understand and represent fractions on a number line diagram.

A. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

B. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

**Assessment Boundary:** Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

The **Basic** student is able to represent fractions on a number line diagram when given a number line pre-partitioned into equal parts from 0 to 1.

A. Represent a fraction $\frac{1}{b}$ on a number line diagram. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

B. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

**Assessment Boundary:** Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

The **Below Basic** student may be able to represent fractions on a number line diagram when given a number line pre-partitioned into equal parts from 0 to 1 with partial success.

A. Represent a fraction $\frac{1}{b}$ on a number line diagram. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

B. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

**Assessment Boundary:** Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

**3.NF.F.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.**

**3.NF.F.3A** Understand two fractions as equivalent if they are the same size, or the same point on a number line.

**3.NF.F.3B** Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent.

**3.NF.F.3C** Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

**3.NF.F.3D** Compare two fractions with the same numerator or the same denominator, by reasoning about their size, Recognize that valid comparisons rely on the two fractions referring to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.

In addition to Proficient, the **Advanced** student is able to compare fractions with different numerators or different denominators, by reasoning about their size, recognize that valid comparisons rely on the two fractions referring to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.

**Assessment Boundary:** Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

The **Proficient** student is able to explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

A. Understand two fractions as equivalent if they are the same size, or the same point on a number line.

B. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent.

C. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

D. Compare two fractions with the same numerator or the same denominator, by reasoning about their size. Recognize that valid comparisons rely on the two fractions referring to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.

**Assessment Boundary:** Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

The **Basic** student is able to explain equivalence of fractions in special cases, and compare fractions by reasoning about their size when given a pictorial representation.

A. Understand two fractions as equivalent if they are the same size, or the same point on a number line.
B. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent.
C. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
D. Compare two fractions with the same numerator or the same denominator, by reasoning about their size. Recognize that valid comparisons rely on the two fractions referring to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.

**Assessment Boundary:** Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

The **Below Basic** student may be able to explain equivalence of fractions in special cases, and compare fractions by reasoning about their size with partial success when given a pictorial representation.

A. Understand two fractions as equivalent if they are the same size, or the same point on a number line.
B. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent.
C. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
D. Compare two fractions with the same numerator or the same denominator, by reasoning about their size. Recognize that valid comparisons rely on the two fractions referring to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.

**Assessment Boundary:** Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

## MEASUREMENT AND DATA

### Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

**3.MD.G.1 Use analog clocks to tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes.**

In addition to Proficient, the **Advanced** student is able to:

- Use analog clocks to tell and write time to the nearest minute and measure time intervals in minutes. Solve multi-step word problems involving addition and subtraction of time intervals in minutes. **OR**
- Solve multi-step word problems involving multiplication of time intervals in minutes. **OR**
- Solve multi-step word problems involving elapsed time.

The **Proficient** student is able to use analog clocks to tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes.

The **Basic** student is able to use analog clocks to tell and write time to the nearest 5 minute intervals. Solve word problems involving addition and subtraction of time intervals of 5 minutes.

The **Below Basic** student may be able to use analog clocks to tell and write time to the nearest 5 minute intervals with partial success. Solve word problems involving addition and subtraction of time intervals of 5 minutes with partial success.

**3.MD.G.2 Measure and estimate liquid volumes and masses of objects using grams (g), kilograms (kg), and liters (L). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. (Excludes multiplicative comparison problems involving notions of "times as much.")**

In addition to Proficient, the **Advanced** student is able to:

- Solve one-step problems involving liquid measures and masses using the four operations requiring reading a measurement off of a scaled measurement tool. **OR**
- Estimate the combined mass or volume of real-world objects or amounts with relative accuracy and calculate the actual mass or volume to determine accuracy.

The **Proficient** student is able to measure and estimate liquid volumes and masses of objects using grams (g), kilograms (kg), and liters (L). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. (Excludes multiplicative comparison problems involving notions of "times as much.")

The **Basic** student is able to measure and estimate liquid volumes and masses of objects using grams (g), kilograms (kg), and liters (L). (Excludes compound units such as cm^3 and finding the geometric volume of a container and excludes multiplicative comparison problems involving notions of "times as much.")

**Assessment Boundary:** Whole number measurements only.

The **Below Basic** student may be able to measure and estimate liquid volumes and masses of objects using grams (g), kilograms (kg), and liters (L) with partial success. (Excludes compound units such as cm^3 and finding the geometric volume of a container and excludes multiplicative comparison problems involving notions of "times as much.")

**Assessment Boundary:** Whole number measurements only.

## Represent and interpret data.

**3.MD.H.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled graphs.**

In addition to Proficient, the **Advanced** student is able to:

- Interpret how changes in the design of a graph can alter impressions of the data it represents **OR**
- Use a scaled picture graph and a scaled bar graph to solve multi-step problems.

The **Proficient** student is able to draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled graphs.

The **Basic** student is able to solve one- and two-step "how many more" and "how many less" problems using information presented in scaled picture graphs and scaled bar graphs.

The **Below Basic** student may be able to solve one- and two-step "how many more" and "how many less" problems using information presented in scaled picture graphs and scaled bar graphs with partial success.

**3.MD.H.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Use the data to create a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.**

In addition to Proficient, the **Advanced** student is able to generate measurement data by measuring lengths using rulers marked with halves, fourths, and eighths of an inch. Use the data to create a line plot marking it in appropriate units—whole numbers, halves, quarters, or eighths.
The **Proficient** student is able to generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Use the data to create a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.
The **Basic** student is able to use given data to create a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.
The **Below Basic** student may be able to use given data to create a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters with partial success.

## Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

**3.MD.I.5 Understand area as an attribute of plane figures and understand concepts of area measurement, such as square units without gaps or overlaps.**

In addition to Proficient, the **Advanced** student is able to create a real-world scenario using area concepts.
The **Proficient** student is able to identify area as an attribute of plane figures and apply concepts of area measurement, such as square units without gaps or overlaps.
The **Basic** student is able to identify concepts of area measurement to determine appropriate representations, when given appropriate and inappropriate representations of area of a plane figure.
The **Below Basic** student may be able to identify concepts of area measurement to determine appropriate representations, with partial success, when given appropriate and inappropriate representations of area of a plane figure.

**3.MD.I.6 Measure areas by counting unit squares (square cm, square m, square in., square ft., and improvised units).**

In addition to Proficient, the **Advanced** student is able to:

- Estimate areas of non-rectangular polygons or find areas of combined rectangular figures by counting unit squares and labeling in appropriate units (square cm, square m, square in, square ft, and improvised units).
- Find the area of a rectangle using a non-counting strategy.

The **Proficient** student is able to measure area by counting unit squares and labeling in appropriate units (square cm, square m, square in, square ft, and improvised units).
The **Basic** student is able to measure area by tiling with unit squares and labeling in square units.
The **Below Basic** student may be able to measure area by tiling with unit squares and labeling in square units with partial success.

**3.MD.I.7 Relate area to the operations of multiplication and addition.**

**3.MD.I.7A** Find the area of a rectangle with whole-number side lengths (dimensions) by multiplying them. Show that this area is the same as when counting unit squares.

**3.MD.I.7B** Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

**3.MD.I.7C** Use area models to represent the Distributive Property in mathematical reasoning. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths $a$ and $b + c$ is the sum of $a \cdot b$ and $a \cdot c$.

In addition to Proficient, the **Advanced** student is able to solve for a missing dimension or partial dimension when given area and one dimension and write an equation to support their thinking.

The **Proficient** student is able to relate area to the operations of multiplication and addition.

A. Find the area of a rectangle with whole-number side lengths (dimensions) by multiplying them. Show that this area is the same as when counting unit squares.

B. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

C. Use area models to represent the Distributive Property in mathematical reasoning. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths $a$ and $b + c$ is the sum of $a \cdot b$ and $a \cdot c$.

The **Basic** student is able to Measure area by tiling with unit squares.

A. Find the area of a rectangle with whole-number side lengths (dimensions) by tiling them or using a pre-partitioned rectangle.

B. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning when given tiling or pre-partitioned rectangles.

The **Below Basic** student may be able to Measure area by tiling with unit squares.

A. Find the area of a rectangle with whole-number side lengths (dimensions) by tiling them or using a pre-partitioned rectangle with partial success.

B. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning with partial success when given tiling or pre-partitioned rectangles.

## Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

**3.MD.J.8 Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different area or with the same area and different perimeter.**

In addition to Proficient, the **Advanced** student is able to:

- Build a rectangle or polygon with given perimeter. **OR**
- Find multiple possible perimeters of a rectangle with a given area. **OR**
- Find multiple possible areas of a rectangle with a given perimeter.

The **Proficient** student is able to solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different area or with the same area and different perimeter.

The **Basic** student is able to solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, and finding an unknown side length when given a pictorial representation.
The **Below Basic** student may be able to solve real-world and mathematical problems involving perimeters of polygons, including:

- Finding the perimeter, when given the side lengths, **AND**
- Finding an unknown side length, when given a pictorial representation.

## GEOMETRY

### Reason with shapes and their attributes.

**3.G.K.1 Use attributes of quadrilaterals to classify rhombuses, rectangles, and squares. Understand that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.**

In addition to Proficient, the **Advanced** student is able to generate possible categories and subcategories to classify and/or group polygons.
The **Proficient** student is able to use attributes of quadrilaterals to classify rhombuses, rectangles, and squares. Understand that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
The **Basic** student is able to select attributes that correspond to a given quadrilateral or select quadrilaterals that correspond to given attributes.
The **Below Basic** student may be able to select attributes that correspond to a given quadrilateral or select quadrilaterals that correspond to given attributes with partial success.

**3.G.K.2 Partition rectangles, regular polygons, and circles into parts with equal areas. Express the area of each part as a unit fraction of the whole.**

In addition to Proficient, the **Advanced** student is able to use a unit fraction to represent a fraction larger than one by partitioning geometric figure(s).
The **Proficient** student is able to partition rectangles, regular polygons, and circles into parts with equal areas. Express the area of each part as a unit fraction of the whole.
The **Basic** student is able to identify the unit fraction of the whole when given a rectangle, regular polygon, or circle that has been pre-partitioned into equal parts.
The **Below Basic** student may be able to identify the unit fraction of the whole with partial success when given a rectangle, regular polygon, or circle that has been pre-partitioned into equal parts.

**Companion document to the 2018 Mathematics Content Standards**

# Grade 4 Mathematics Content Standards & PLDs

## GRADE 4 MATH PRACTICES

### MP1 Make sense of problems and persevere in solving them.

**4.MP.1** In grade four, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third grade students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense? "They listen to the strategies of others and will try different approaches. They will often use another method to check their answers.

### MP2 Reason abstractly and quantitatively.

**4.MP.2** Students recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.

### MP3 Construct viable arguments and critique the reasoning of others.

**4.MP.3** Students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like, "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.

### MP4 Model with mathematics.

**4.MP.4** Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect different representations and explain the connections. They should be able to use all of these representations as needed. Students should evaluate their results in the context of the situation and reflect on whether the results make sense.

### MP5 Use appropriate tools strategically.

**4.MP.5** Students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals, they may use protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.

### MP6 Attend to precision.

**4.MP.6** As students develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and stating the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.

### MP7 Look for and make use of structure.

**4.MP.7** Students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.

### MP8 Look for and express regularity in repeated reasoning.

**4.MP.8** Students notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

## OPERATIONS AND ALGEBRAIC THINKING

### Use the four operations with whole numbers to solve problems.

**4.OA.A.1 This standard was Intentionally removed by the 2018 Math Standards Review Committee.**

**4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, by using strategies including, but not limited to, drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.**

In addition to Proficient, the **Advanced** student is able to recognize how many times larger one quantity is than another, when comparing two quantities.
The **Proficient** student is able to multiply or divide to solve word problems involving multiplicative comparison, by using strategies including, but not limited to, drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
The **Basic** student is able to multiply or divide to solve word problems involving multiplicative comparison when given an equation or pictorial representation for the problem.
The **Below Basic** student may be able to multiply or divide to solve word problems involving multiplicative comparison with partial success, when given an equation or pictorial representation for the problem.

**4.OA.A.3 Solve multi-step word problems posed with whole numbers, including problems in which remainders must be interpreted.**

**4.OA.A.3A** Represent these problems using equations with a letter standing for the unknown quantity.
**4.OA.A.3B** Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

In addition to Proficient, the **Advanced** student is able to create a real-world problem when given a division problem that includes a remainder, then solve and interpret the remainder.
The **Proficient** student is able to solve multi-step word problems posed with whole numbers, including problems in which remainders must be interpreted.
A. Represent these problems using equations with a letter standing for the unknown quantity.
B. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
The **Basic** student is able to solve one-step word problems posed with whole numbers, including problems in which remainders must be interpreted.
A. Represent these problems using equations with a letter standing for the unknown quantity.
B. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
The **Below Basic** student may be able to solve one-step word problems posed with whole numbers with partial success, including problems in which remainders must be interpreted.
A. Represent these problems using equations with a letter standing for the unknown quantity.

B. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

## Develop understanding of factors and multiples.

**4.OA.B.4 Demonstrate an understanding of factors and multiples.**

**4.OA.B.4A** Find all factor pairs for a whole number in the range 1-100.
**4.OA.B.4B** Recognize that a whole number is a multiple of each of its factors.
**4.OA.B.4C** Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number.
**4.OA.B.4D** Determine whether a given whole number in the range 1-100 is prime or composite.

In addition to Proficient, the **Advanced** student is able to find the prime factorization of a given number within the range of 1-100.

The **Proficient** student is able to demonstrate an understanding of factors and multiples.

A. Find all factor pairs for a whole number in the range 1-100.
B. Recognize that a whole number is a multiple of each of its factors.
C. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number.
D. Determine whether a given whole number in the range 1-100 is prime or composite.

The **Basic** student is able to demonstrate an understanding of factors and multiples.

A. Find all factor pairs for a whole number in the range 1-25.
B. Recognize that a whole number is a multiple of each of its factors.
C. Determine whether a given whole number in the range 1-25 is a multiple of a given one-digit number.
D. Determine whether a given whole number in the range 1-25 is prime or composite.

The **Below Basic** student may be able to demonstrate an understanding of factors and multiples with partial success.

A. Find all factor pairs for a whole number in the range 1-25.
B. Recognize that a whole number is a multiple of each of its factors.
C. Determine whether a given whole number in the range 1-25 is a multiple of a given one-digit number.
D. Determine whether a given whole number in the range 1-25 is prime or composite.

## Generate and analyze patterns.

**4.OA.C.5 Given a pattern, explain a rule that the pattern follows and extend the pattern. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.**

In addition to Proficient, the **Advanced** student is able to generate a pattern, write a rule, and predict a term in a number or shape pattern.

The **Proficient** student is able to, given a pattern, explain the rule that the pattern follows and extend the pattern. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

The **Basic** student is able to, given a pattern and the rule that the pattern follows, extend the pattern.

The **Below Basic** student may be able to, given a pattern and the rule that the pattern follows, extend the pattern with partial success.

## NUMBER AND OPERATIONS IN BASE TEN

### Generalize place value understanding for multi-digit whole numbers (limited to numbers less than or equal to 1,000,000).

**4.NBT.D.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.**

In addition to Proficient, the **Advanced** student is able to:

- Recognize that a digit in one place represents a multiple of 10 times what another digit represents in the place to the right and apply this relationship as an equation. **OR**
- Compare numbers beyond millions by reasoning about place value. **OR**
- Extend reasoning about place value to go beyond one place value to the right.

The **Proficient** student is able to recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

The **Basic** student is able to recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right when given a visual model or representation.

The **Below Basic** student may be able to recognize, that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right with partial success, when given a visual model or representation.

**4.NBT.D.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols.**

In addition to Proficient, the **Advanced** student is able to apply understanding of numbers beyond 1,000,000 to a real-world context or situation and compare them using >, =, and < symbols.

The **Proficient** student is able to read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols.

The **Basic** student is able to read and write multi-digit whole numbers up to 10,000 using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols.

The **Below Basic** student may be able to read and write multi-digit whole numbers up to 10,000 using base-ten numerals, number names, and expanded form with partial success. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols with partial success.

**4.NBT.D.3 Use place value understanding to round multi-digit whole numbers to any place.**

In addition to Proficient, the **Advanced** student is able to:

- Explain how to use the digits in multi-digit whole numbers to round numbers up to 1,000,000. **OR**
- Use an example of rounding and explain how it is helpful in computation. **OR**
- Justify the appropriate place value to which the student would round in a given situation.

The **Proficient** student is able to use place value understanding to round multi-digit whole numbers up to 1,000,000 to any place.

The **Basic** student is able to use place value understanding to round multi-digit whole numbers up to 1,000,000 to any place when provided a model such as a number line with benchmark numbers.

The **Below Basic** student may be able to use place value understanding to round multi-digit whole numbers up to 1,000,000 to any place, with partial success, when provided a model such as a number line with benchmark numbers.

## Use place value understanding and properties of operations to perform multi-digit arithmetic (limited to whole numbers less than or equal to 1,000,000).

**4.NBT.E.4 Add and subtract multi-digit whole numbers using place value strategies including the standard algorithm.**

In addition to Proficient, the **Advanced** student is able to extend the standard algorithm for addition and subtraction of whole numbers to the addition and subtraction of decimal values.
The **Proficient** student is able to add and subtract multi-digit whole numbers using place value strategies including the standard algorithm.
The **Basic** student is able to add or subtract two or more numbers, using place value strategies including the standard algorithm, whose sum or difference is less than 10,000.
The **Below Basic** student may be able to add or subtract two or more numbers, using place value strategies, with partial success, including the standard algorithm, whose sum or difference is less than 10,000.

**4.NBT.E.5 Use strategies based on place value and the properties of multiplication.**

**4.NBT.E.5A** Multiply a whole number of up to four digits by a one-digit whole number.
**4.NBT.E.5B** Multiply a pair of two-digit numbers.
**4.NBT.E.5C** Use appropriate models to explain the calculation, such as by using equations, rectangular arrays, and/or area models.

In addition to Proficient, the **Advanced** student is able to multiply a whole number of up to four digits by a one-digit whole number or a pair of two-digit numbers using more than one model or strategy and defend the efficiency of the strategy used.
The **Proficient** student is able to use strategies based on place value and the properties of multiplication to:

A. Multiply a whole number of up to four digits by a one-digit whole number.
B. Multiply a pair of two-digit numbers.
C. Use appropriate models to explain the calculation, such as by using equations, rectangular arrays, ratio tables, or area models.

The **Basic** student is able to use strategies based on place value and the properties of multiplication to:

A. Multiply a whole number of up to four digits by a one-digit whole number when given a partially completed model.
B. Multiply a pair of two-digit numbers when given a partially completed model.

The **Below Basic** student may be able to use strategies based on place value and the properties of multiplication with partial success to:

A. Multiply a whole number of up to four digits by a one-digit whole number when given a partially completed model.
B. Multiply a pair of two-digit numbers when given a partially completed model.

**4.NBT.E.6 Use strategies based on place value, the properties of multiplication, and/or the relationship between multiplication and division to find quotients and remainders with up to four-digit dividends and one-digit divisors. Use appropriate models to explain the calculation, such as by using equations, rectangular arrays, and/or area models.**

In addition to Proficient, the **Advanced** student is able to create a real-world situation that can be modeled using a given division problem **or** find quotients and remainders with up to four-digit dividends and one-digit divisors using more than one model or strategy and defend the efficiency of the strategy used.
The **Proficient** student is able to use strategies based on place value, the properties of multiplication, and/or the relationship between multiplication and division to find quotients and remainders with up to four-digit dividends and one-digit divisors. Use appropriate models to explain the calculation, such as by using equations, rectangular arrays, ratio tables, or area models.

The **Basic** student is able to use partially completed models to find quotients and remainders with up to four-digit dividends and one-digit divisors.
The **Below Basic** student may be able to use partially completed models to find quotients and remainders with up to four-digit dividends and one-digit divisors with partial success.

## NUMBER AND OPERATIONS - FRACTIONS

### Extend understanding of fraction equivalence and ordering (limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100).

**4.NF.F.1 Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{n \cdot a}{n \cdot b}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.**

In addition to Proficient, the **Advanced** student is able to use knowledge of equivalent fractions to solve real-world problems.
The **Proficient** student is able to explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{n \cdot a}{n \cdot b}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
The **Basic** student is able to identify equivalent fractions when given visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size.
The **Below Basic** student may be able to identify equivalent fractions, with partial success, when given visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size.

**4.NF.F.2 Compare two fractions with different numerators and different denominators by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$.**

**4.NF.F.2A** Recognize that comparisons are valid only when the two fractions refer to the same whole.
**4.NF.F.2B** Record the results of comparisons with symbols >, =, or <.
**4.NF.F.2C** Justify the conclusions by using a visual fraction model.

In addition to Proficient, the **Advanced** student is able to compare more than two fractions with different numerators and different denominators by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$.

A. Recognize that comparisons are valid only when the fractions refer to the same whole.
B. Record the results of comparisons on a number line.
C. Justify the conclusions by using a visual fraction model.

**Assessment Boundary:** Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8, 10, 12, and 100.

The **Proficient** student is able to compare two fractions with different numerators and different denominators by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$.

A. Recognize that comparisons are valid only when the two fractions refer to the same whole.
B. Record the results of comparisons with symbols >, =, or <.
C. Justify the conclusions by using a visual fraction model.

**Assessment Boundary:** Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8, 10, 12, and 100.

The **Basic** student is able to compare two fractions with different numerators and different denominators by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$ when provided pre-partitioned visual models.

A. Recognize that comparisons are valid only when the two fractions refer to the same whole.
B. Record the results of comparisons with symbols >, =, or <.

**Assessment Boundary:** limit fractions to have denominators 2, 3, 4, 6, 8, 10, and 100.

The **Below Basic** student may be able to compare two fractions with different numerators and different denominators by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$, with partial success, when provided pre-partitioned visual models.

A. Recognize that comparisons are valid only when the two fractions refer to the same whole.
B. Record the results of comparisons with symbols >, =, or <.

**Assessment Boundary:** limit fractions to have denominators 2, 3, 4, 6, 8, 10, and 100.

## Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers (limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100).

**4.NF.G.3 Understand a fraction a/b with $a > 1$ as a sum of unit fractions (1/b).**

**4.NF.G.3A** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

**4.NF.G.3B** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions by using a visual fraction model.

**4.NF.G.3C** Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction, and/or by using properties of addition and the relationship between addition and subtraction.

**4.NF.G.3D** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.

In addition to Proficient, the **Advanced** student is able to:

A-B. Use properties of operations and inverse operations to add or subtract two fractions with like denominators including mixed numbers.
C. Identify and represent addition and subtraction of fractions with like denominators in multiple ways.
D. Solve two-step problems involving addition or subtraction of fractions with like denominators in mathematical or real-world contexts.

The **Proficient** student is able to understand a fraction a/b with $a > 1$ as a sum of unit fractions ($\frac{1}{b}$).

A. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
B. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions by using a visual fraction model.
C. Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction, and/or by using properties of addition and the relationship between addition and subtraction.
D. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.

The **Basic** student is able to with a pictorial representation:

A-B. Interpret a fraction as a sum or a difference of unit fractions.
C. Convert mixed numbers into equivalent fractions.
D. Solve one-step problems involving addition or subtraction of fractions with like denominators in mathematical contexts.

The **Below Basic** student may be able to, with a pictorial representation:

A-B. Interpret a fraction as a sum or a difference of unit fractions with partial success.
C. Convert mixed numbers into equivalent fractions with partial success.
D. Solve one-step problems involving addition or subtraction of fractions with like denominators in mathematical contexts with partial success.

**4.NF.G.4 Apply and extend an understanding of multiplication by multiplying a whole number and a fraction.**

**4.NF.G.4A** Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$.

**4.NF.G.4B** Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number.

**4.NF.G.4C** Solve real-world problems involving multiplication of a fraction by a whole number, using visual fraction models and equations to represent the problem.

In addition to Proficient, the **Advanced** student is able to generalize and explain the multiplication of a whole number and a fraction as $nx\ (a/b) = (nx \cdot a)/b$ by creating a visual fraction model in the context of a real-world problem.

The **Proficient** student is able to apply and extend an understanding of multiplication by multiplying a whole number and a fraction.

A. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$.

B. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number.

C. Solve real-world problems involving multiplication of a fraction by a whole number, using visual fraction models and equations to represent the problem.

The **Basic** student is able to apply and extend an understanding of multiplication by multiplying a whole number and a fraction when given a pictorial representation.

A. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$.

B. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number.

C. Solve real-world problems involving multiplication of a fraction by a whole number, given visual fraction models and equations to represent the problem.

The **Below Basic** student may be able to apply and extend an understanding of multiplication by multiplying a whole number and a fraction when given a pictorial representation.

A. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ with partial success.

B. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number with partial success.

C. Solve real-world problems involving multiplication of a fraction by a whole number, with partial success, when given visual fraction models and equations to represent the problem.

## Understand decimal notation for fractions, and compare decimal fractions.

**4.NF.H.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.**

In addition to Proficient, the **Advanced** student is able to express a fraction with denominator 10 as an equivalent fraction with denominator 100 with fractions greater than one whole, and use this technique to add two fractions with respective denominators 10 and 100.

The **Proficient** student is able to express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.

The **Basic** student is able to express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100 when given a visual model.

The **Below Basic** student may be able to express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100 with partial success when given a visual model.

**4.NF.H.6 Use decimal notation for fractions with denominators 10 or 100.**

In addition to Proficient, the **Advanced** student is able to:

- Apply decimal notation for fractions with denominators 10 or 100 with fractions greater than one whole. **OR**
- Apply decimal notation for fractions with a denominator of 1000.

The **Proficient** student is able to apply decimal notation for fractions with denominators 10 or 100.
The **Basic** student is able to apply decimal notation for fractions with denominators 10 or 100 when given a visual model.
The **Below Basic** student may be able to apply decimal notation for fractions with denominators 10 or 100 with partial success when given a visual model.

**4.NF.H.7 Compare and order decimal numbers to hundredths and justify by using concrete and visual models. Record the results of comparisons with the words "is greater than," "is equal to," "is less than," and with the symbols >, =, and <.**

In addition to Proficient, the **Advanced** student is able to compare and order decimal numbers greater than one and/or to thousandths and justify by using concrete and visual models. Record the results of comparisons with the words "is greater than," "is equal to," "is less than," and with the symbols >, =, and <.
The **Proficient** student is able to compare and order decimal numbers to hundredths and justify by using concrete and visual models. Record the results of comparisons with the words "is greater than," "is equal to," "is less than," and with the symbols >, =, and <.
The **Basic** student is able to compare and order decimal numbers to hundredths when given visual models. Record the results of comparisons with the words "is greater than," "is equal to," "is less than," and with the symbols >, =, and <.
The **Below Basic** student may be able to compare and order decimal numbers to hundredths with partial success when given visual models. Record the results of comparisons with the words "is greater than," "is equal to," "is less than," and with the symbols >, =, and < with partial success.

## MEASUREMENT AND DATA

### Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

**4.MD.I.1 Know relative sizes of measurement units within one system of units including, but not limited to, km, m, cm; kg, g; lb., oz.; l L, ml; hr., min, sec; ft., in., gal., qt. pt., c. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.**

In addition to Proficient, the **Advanced** student is able to estimate a given unit of measurement from one system to another (i.e., km vs. miles, kg vs lbs, etc.).
The **Proficient** student is able to know relative sizes of measurement units within one system of units including, but not limited to, km, m, cm; kg, g; lb, oz.; l L, ml; hr, min, sec; ft., in., gal., qt. pt., c. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.
The **Basic** student is able to, within a single system of measurement, express measurements in a larger unit in terms of a smaller unit when given a labeled model or two-column table.
The **Below Basic** student may be able to, within a single system of measurement, express measurements in a larger unit in terms of a smaller unit with partial success when given a labeled model or two-column table.

**4.MD.I.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.**

**Assessment Boundary:** Use denominators of 2, 4, 8 and decimals up to hundredths.

In addition to Proficient, the **Advanced** student is able to create and solve real-world scenarios using the four operations involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit or smaller units in terms of larger units. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

**Assessment Boundary:** Use denominators of 2, 4, 8 and decimals up to hundredths.

The **Proficient** student is able to use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

**Assessment Boundary:** Use denominators of 2, 4, 8 and decimals up to hundredths.

The **Basic** student is able to use the four operations to solve one-step word problems (involving distances, intervals of time, liquid volumes, masses of objects, and money) which require expressing measurements given in a larger unit in terms of a smaller unit when given diagrams that feature a measurement scale.

**Assessment Boundary:** Use denominators of 2, 4, 8 and decimals up to hundredths.

The **Below Basic** student may be able to use the four operations to solve one-step word problems (involving distances, intervals of time, liquid volumes, masses of objects, and money) which require expressing measurements given in a larger unit in terms of a smaller unit with partial success, when given diagrams that feature a measurement scale.

**Assessment Boundary:** Use denominators of 2, 4, 8 and decimals up to hundredths.

**4.MD.I.3 Apply the area and perimeter formulas for rectangles in real-world and mathematical problems.**

In addition to Proficient, the **Advanced** student is able to:

- Apply the formula for area of a rectangle to find the area of a right triangle. **OR**
- Find the largest possible area of a rectangle when given a specific perimeter. **OR**
- Find the largest possible perimeter of a rectangle when given a specific area.

The **Proficient** student is able to apply the area and perimeter formulas for rectangles in real-world and mathematical problems.

The **Basic** student is able to determine the area and perimeter of rectangles when given a labeled (length and width) pictorial representation.

The **Below Basic** student may be able to determine the area and perimeter of rectangles with partial success when given a labeled (length and width) pictorial representation.

## Represent and interpret data.

**4.MD.J.4 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots.**

In addition to Proficient, the **Advanced** student is able to

- Solve problems involving multiplication of fractions by using information presented in line plots. **AND**
- Use multiplicative thinking to generalize data from a line plot.

The **Proficient** student is able to make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots.

The **Basic** student is able to make a line plot to:

- Display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). **OR**
- Solve problems involving addition and subtraction of fractions by using information presented in given line plots.

The **Below Basic** student may be able to:

- Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$) with partial success. **OR**
- Solve problems involving addition and subtraction of fractions by using information presented in given line plots with partial success.

## Geometric measurement: understand concepts of angle and measure angles.

**4.MD.K.5**

**4.MD.K.5A Regarding angles, recognize angles as geometric shapes that are formed wherever two rays share a common endpoint.**

**4.MD.K.5B Regarding angles, understand concepts of angle measurement. An angle is measured with reference to a circle with its center at the common endpoint of the rays.**

In addition to Proficient, the **Advanced** student is able to apply understanding of angles as geometric shapes and find examples of angles in the real world for different angle measurements.

The **Proficient** student is able to:

A. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint.

B. Understand concepts of angle measurement. An angle is measured with reference to a circle with its center at the common endpoint of the rays.

The **Basic** student is able to:

A. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint. **OR**

B. Understand concepts of angle measurement. An angle is measured with reference to a circle with its center at the common endpoint of the rays.

The **Below Basic** student may be able to:

A. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint with partial success. **OR**

B. Understand concepts of angle measurement with partial success. An angle is measured with reference to a circle with its center at the common endpoint of the rays.

**4.MD.K.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.**

In addition to Proficient, the **Advanced** student is able to measure angles in whole-number degrees greater than 180 degrees using a protractor. Sketch angles of specified measure in whole-number degrees greater than 180 degrees.
The **Proficient** student is able to measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
The **Basic** student is able to:

- Measure angles in whole-number degrees using a protractor. **OR**
- Sketch angles of specified measure.

The **Below Basic** student may be able to:

- Measure angles in whole-number degrees with partial success using a protractor. **OR**
- Sketch angles of specified measure with partial success.

**4.MD.K.7 Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.**

In addition to Proficient, the **Advanced** student is able to:

- Solve addition and subtraction problems to find more than one unknown angle (may include angles greater than 180 degrees) on a diagram. **OR**
- Create a diagram from a real-world problem to find unknown angles.

The **Proficient** student is able to solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.
The **Basic** student is able to solve addition and subtraction problems to find unknown angles on a diagrams where the sum of the angles is 90 or 180 degrees in real-world and mathematical problems.
The **Below Basic** student may be able to solve addition and subtraction problems to find unknown angles on a diagrams where the sum of the angles is 90 or 180 degrees in real-world and mathematical problems with partial success.

## GEOMETRY

### Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

**4.G.L.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.**

In addition to Proficient, the **Advanced** student is able to create a two-dimensional shape when given specific attributes.
The **Proficient** student is able to draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
The **Basic** student is able to identify points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines in two-dimensional figures.
The **Below Basic** student may be able to identify points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines in two-dimensional figures with partial success.

**4.G.L.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.**

In addition to Proficient, the **Advanced** student is able to create two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of specified size; classify triangles and justify their reasoning.

The **Proficient** student is able to classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category and identify right triangles.

The **Basic** student is able to:

- Identify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. **OR**
- Recognize right triangles as a category and identify right triangles.

The **Below Basic** student may be able to:

- Identify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size with partial success. **OR**
- Recognize right triangles as a category and identify right triangles with partial success.

**4.G.L.3 Identify line-symmetric figures. Recognize and draw lines of symmetry for two-dimensional figures.**

In addition to Proficient, the **Advanced** student is able to create a figure with a given number of lines of symmetry.

The **Proficient** student is able to identify line-symmetric figures. Recognize and draw lines of symmetry for two-dimensional figures.

The **Basic** student is able to:

- Identify line-symmetric figures. **OR**
- Recognize and draw lines of symmetry for two-dimensional figures.

The **Below Basic** student may be able to:

- Identify line-symmetric figures with partial success **OR**
- Recognize and draw lines of symmetry for two-dimensional figures with partial success.

**Companion document to the 2018 Mathematics Content Standards**

# Grade 5 Mathematics Content Standards & PLDs

## GRADE 5 MATH PRACTICES

### MP1 Make sense of problems and persevere in solving them.

**5.MP.1** In grade five, students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"

### MP2 Reason abstractly and quantitatively.

**5.MP.2** Students recognize that a number represents a specific quantity. They connect quantities to written symbols and create logical representation of the problem at hand, while considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.

### MP3 Construct viable arguments and critique the reasoning of others.

**5.MP.3** Students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like, "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.

### MP4 Model with mathematics.

**5.MP.4** Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, to create equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.

### MP5 Use appropriate tools strategically.

**5.MP.5** Fifth graders consider the available tools, including estimation, when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems, or to make predictions from real-world data.

### MP6 Attend to precision.

**5.MP.6** Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about

specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism, they record their answers in cubic units.

### MP7 Look for and make use of structure.

**5.MP.7** Students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.

### MP8 Look for and express regularity in repeated reasoning.

**5.MP.8** Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and to perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

## OPERATIONS AND ALGEBRAIC THINKING

### Write, interpret, and/or evaluate numerical expressions.

**5.OA.A.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.**

In addition to Proficient, the **Advanced** student is able to create and evaluate numerical expressions that use two or more types of grouping symbols to complete the simplification of numerical expressions.
The **Proficient** student is able to use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
The **Basic** student is able to evaluate two-step numerical expressions with no grouping symbols.
The **Below Basic** student may be able to evaluate two-step numerical expressions with no grouping symbols with partial success.

**5.OA.A.2 Write simple expressions requiring parentheses that record calculations with numbers, and interpret numerical expressions without evaluating them.**

In addition to Proficient, the **Advanced** student is able to, using the order and properties of operations, determine whether expressions are equivalent and justify their thinking.
The **Proficient** student is able to write simple expressions requiring parentheses that record calculations with numbers, and interpret numerical expressions without evaluating them.
The **Basic** student is able to:

- Write simple expressions requiring parentheses that record calculations with numbers. **OR**
- Interpret numerical expressions without evaluating them.

The **Below Basic** student may be able to:

- Write simple expressions requiring parentheses that record calculations with numbers with partial success. **OR**
- Interpret numerical expressions without evaluating them with partial success.

### Analyze patterns and relationships.

**5.OA.B.3 Generate two numerical patterns with each pattern having its own rule. Explain informally the relationship(s) between corresponding terms in the two patterns.**

**5.OA.B.3A** Form ordered pairs consisting of corresponding terms from the two patterns.

**5.OA.B.3B** Graph the ordered pairs on a coordinate plane.

In addition to Proficient, the **Advanced** student is able to identify and explain features between the corresponding terms of two numerical patterns not explicitly given in the rule.

The **Proficient** student is able to generate two numerical patterns with each pattern having its own rule. Explain informally the relationship(s) between corresponding terms in the two patterns.

A. Form ordered pairs consisting of corresponding terms from the two patterns.

B. Graph the ordered pairs on a coordinate plane.

The **Basic** student is able to extend two numerical patterns with each pattern having its own rule. Form ordered pairs consisting of corresponding terms from the two patterns.

The **Below Basic** student may be able to extend two numerical patterns with each pattern having its own rule with partial success. Form ordered pairs consisting of corresponding terms from the two patterns with partial success.

## NUMBER AND OPERATIONS IN BASE TEN

### Understand the place value system.

**5.NBT.C.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.**

In addition to Proficient, the **Advanced** student is able to:

- Recognize that given two different digits in a multi-digit number, one digit can represent a multiple of 100 times the digit two places to its right, and a multiple of $\frac{1}{100}$ times the digit two places to its left. **OR**
- Extend reasoning about place value to go two or more places to the left and right.

The **Proficient** student is able to recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.

The **Basic** student is able to recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left when given a visual model or representation.

The **Below Basic** student may be able to recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left with partial success, when given a visual model or representation.

**5.NBT.C.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10.**

In addition to Proficient, the **Advanced** student is able to:

- Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10 and use whole number exponents to denote powers of 10. **OR**
- Compare two powers of 10 expressed exponentially. **OR**
- Apply understanding of exponents to real-world examples of scientific notation.

The **Proficient** student is able to explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10.
The **Basic** student is able to continue a pattern of a number multiplied and divided by a power of 10.
The **Below Basic** student may be able to continue a pattern of a number multiplied and divided by a power of 10 with partial success.

**5.NBT.C.3 Read, write, and compare decimals to thousandths.**

**5.NBT.C.3A** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.
**5.NBT.C.3B** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols.

In addition to Proficient, the **Advanced** student is able to:

- Relate decimals to real-world scenarios with scientific notation. **OR**
- Create numbers in different forms, order them, and justify reasoning for order. **OR**
- Apply what is known about comparing decimals to be able to analyze and justify errors in comparisons.

The **Proficient** student is able to:

A. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.
B. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols.

The **Basic** student is able to:

A. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. **OR**
B. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols.

The **Below Basic** student may be able to:

A. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form with partial success. **OR**
B. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols with partial success.

**5.NBT.C.4 Use place value understanding to round decimals to any place to a given place.**

**Assessment Boundary:** Limit place value to the thousandths.

In addition to Proficient, the **Advanced** student is able to:

- Explain how to use the digits in multi-digit decimal numbers to round numbers to any place. **OR**
- Use an example of rounding decimals and explain how it is helpful in computation. **OR**
- Justify the appropriate place value to which the student would round in a given situation.

**Assessment Boundary:** Limit place value to the thousandths.
The **Proficient** student is able to use place value understanding to round decimals to any place to a given place.
**Assessment Boundary:** Limit place value to the thousandths.

The **Basic** student is able to use place value understanding to round decimals to any place when provided a model such as a number line with benchmark numbers.
**Assessment Boundary:** Limit place value to the thousandths.
The **Below Basic** student may be able to use place value understanding to round decimals to any place with partial success, when provided a model such as a number line with benchmark numbers.
**Assessment Boundary:** Limit place value to the thousandths.

## Perform operations with multi-digit whole numbers and with decimals to hundredths.

**5.NBT.D.5 Multiply multi-digit whole numbers using place value strategies including the standard algorithm.**

In addition to Proficient, the **Advanced** student is able to justify how the various place value strategies for multiplication relate to the standard algorithm.
The **Proficient** student is able to multiply multi-digit whole numbers using place value strategies including the standard algorithm.
The **Basic** student is able to multiply multi-digit whole numbers using place value strategies.
The **Below Basic** student may be able to multiply multi-digit whole numbers using place value strategies with partial success.

**5.NBT.D.6 Find whole-number quotients with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of multiplication, and/or the relationship between multiplication and division, including the standard algorithm. Use appropriate models to Illustrate and explain the calculation, such as equations, rectangular arrays, and/or area models**.

**Assessment Boundary:** The standard algorithm for division will not be assessed.
In addition to Proficient, the **Advanced** student is able to:

- Create a real-world situation that can be modeled using a given division problem. **OR**
- Find quotients and remainders with up to four-digit dividends and two-digit divisors using more than one model or strategy and defend the efficiency of the strategy used. **OR**
- Justify how the various place value strategies for division relate to the standard algorithm.

**Assessment Boundary:** The standard algorithm for division will not be assessed.
The **Proficient** student is able to find whole-number quotients with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of multiplication, and/or the relationship between multiplication and division, including the standard algorithm. Use appropriate models to Illustrate and explain the calculation, such as equations, rectangular arrays, and/or area models. **Assessment Boundary:** The standard algorithm for division will not be assessed.
The **Basic** student is able to:

- Find whole-number quotients with up to four-digit dividends and two-digit divisors when given a partially completed model. **OR**
- Find quotients with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of multiplication, and/or the relationship between multiplication and division. Use appropriate models to Illustrate and explain the calculation, such as equations, rectangular arrays, and/or area models.

**Assessment Boundary:** The standard algorithm for division will not be assessed.
The **Below Basic** student may be able to:

- Find whole-number quotients with up to four-digit dividends and two-digit divisors with partial success when given a partially completed model. **OR**

- Find quotients with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of multiplication, and/or the relationship between multiplication and division with partial success. Use appropriate models to Illustrate and explain the calculation, such as equations, rectangular arrays, and/or area models with partial success.

**Assessment Boundary:** The standard algorithm for division will not be assessed.

**5.NBT.D.7 Add, subtract, multiply, and divide decimals to hundredths using concrete models or drawings, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; Relate the strategy to a written method and explain the reasoning used.**

In addition to Proficient, the **Advanced** student is able to:

- Explain patterns in the base-ten system when finding sums and differences with decimals and how it is consistent with the standard algorithm.
  **OR**
- Investigate and draw conclusions about decimal placement when finding products and quotients with decimals.

The **Proficient** student is able to add, subtract, multiply, and divide decimals to hundredths using concrete models or drawings, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; Relate the strategy to a written method and explain the reasoning used.

The **Basic** student is able to add, subtract, multiply, and divide decimals to hundredths when given concrete models or drawings, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

The **Below Basic** student may be able to add, subtract, multiply, and divide decimals to hundredths with partial success when given concrete models or drawings, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

## NUMBER AND OPERATIONS - FRACTIONS

### Use equivalent fractions as a strategy to add and subtract fractions.

**5.NF.E.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.**

In addition to Proficient, the **Advanced** student is able to add or subtract three or more fractions with unlike denominators (including mixed numbers) that require regrouping by replacing the given fractions with equivalent fractions and justify the denominators chosen.

The **Proficient** student is able to add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

The **Basic** student is able to add and subtract fractions with unlike denominators by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators when given a visual model.

The **Below Basic** student may be able to add and subtract fractions with unlike denominators by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators, with partial success, when given a visual model.

**5.NF.E.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.**

In addition to Proficient, the **Advanced** student is able to solve real-world problems involving addition or subtraction with at least 3 or more fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

The **Proficient** student is able to solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

The **Basic** student is able to solve one-step mathematical problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, when given visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

The **Below Basic** student may be able to solve one-step mathematical problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators with partial success, when given visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

## Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

**5.NF.F.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers by using visual fraction models or equations to represent the problem.**

In addition to Proficient, the **Advanced** student is able to create a real-world situation and visual model to demonstrate understanding between division of whole numbers and fractions.

The **Proficient** student is able to interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers by using visual fraction models or equations to represent the problem.

The **Basic** student is able to interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$) when given a visual model. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers when given visual fraction models or equations to represent the problem.

The **Below Basic** student may be able to interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$) with partial success, when given a visual model. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, with partial success, when given visual fraction models or equations to represent the problem.

**5.NF.F.4 Extend the concept of multiplication to multiply a fraction or whole number by a fraction.**

**5.NF.F.4A** Recognize the relationship between multiplying fractions and finding the areas of rectangles with fractional side lengths.

**5.NF.F.4B** Interpret multiplication of a fraction by a whole number and a whole number by a fraction and compute the product.

**5.NF.F.4C** Interpret multiplication in which both factors are fractions less than one and compute the product.

In addition to Proficient, the **Advanced** student is able to:

A. Draw a visual representation to show understanding of how to multiply a fraction by a fraction **OR** find a missing fractional side length when given an area and one side length of a rectangle.

B. Estimate the result of multiplying a whole number by a fraction less than one, by a fraction equal to one, or by a fraction greater than one and justify the estimation with a visual model or description. Predict the sizes of the factors based on the product without performing the indicated multiplication.

The **Proficient** student is able to extend the concept of multiplication to multiply a fraction or whole number by a fraction.

A. Recognize the relationship between multiplying fractions and finding the areas of rectangles with fractional side lengths.

B. Interpret multiplication of a fraction by a whole number and a whole number by a fraction and compute the product.

C. Interpret multiplication in which both factors are fractions less than one and compute the product.

The **Basic** student is able to:

A. Recognize the relationship between multiplying fractions and finding the areas of rectangles with fractional side lengths when given a visual model.

B. Interpret multiplication of a fraction by a whole number and a whole number by a fraction and compute the product when given a visual model.

C. Interpret multiplication in which both factors are fractions less than one and compute the product when given a visual model.

The **Below Basic** student may be able to:

A. Recognize the relationship between multiplying fractions and finding the areas of rectangles with fractional side lengths, with partial success, when given a visual model.

B. Interpret multiplication of a fraction by a whole number and a whole number by a fraction and compute the product, with partial success, when given a visual model.

C. Interpret multiplication in which both factors are fractions less than one and compute the product, with partial success, when given a visual model.

**5.NF.F.5 Justify the reasonableness of a product when multiplying with fractions.**

**5.NF.F.5A** Estimate the size of the product based on the size of the two factors.

**5.NF.F.5B** Explain why multiplying a given number by a number greater than 1 (improper fractions, mixed numbers, whole numbers) results in a product larger than the given number.

**5.NF.F.5C** Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number.

**5.NF.F.5D** Explain why multiplying the numerator and denominator by the same number has the same effect as multiplying the fraction by 1.

In addition to Proficient, the **Advanced** student is able to estimate the size of a factor based on the product and the other factor without performing the indicated multiplication and justify the reasonableness of a product when multiplying with fractions by creating a visual model or explanation.

The **Proficient** student is able to justify the reasonableness of a product when multiplying with fractions.

A. Estimate the size of the product based on the size of the two factors.

B. Explain why multiplying a given number by a number greater than 1 (improper fractions, mixed numbers, whole numbers) results in a product larger than the given number.

C. Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number.
D. Explain why multiplying the numerator and denominator by the same number has the same effect as multiplying the fraction by 1.

The **Basic** student is able to estimate the size of the product based on the size of the two factors.
The **Below Basic** student may be able to estimate the size of the product based on the size of the two factors with partial success.

**5.NF.F.6 Solve real-world problems involving multiplication of fractions and mixed numbers by using visual fraction models or equations to represent the problem.**

In addition to Proficient, the **Advanced** student is able to:

- Solve multi-step real-world problems involving multiplication of fractions including mixed numbers with multiple strategies or representations. **OR**
- Create and solve real-world problems involving multiplication of fractions and mixed numbers by creating visual fraction models or equations to represent the problem.

The **Proficient** student is able to solve real-world problems involving multiplication of fractions and mixed numbers by using visual fraction models or equations to represent the problem.
The **Basic** student is able to solve real-world problems involving multiplication of fractions and whole numbers when given visual fraction models or equations to represent the problem.
The **Below Basic** student may be able to solve real-world problems involving multiplication of fractions and whole numbers, with partial success, when given visual fraction models or equations to represent the problem.

**5.NF.F.7 Extend the concept of division to divide unit fractions and whole numbers by using visual fraction models and equations.**

**5.NF.F.7A** Interpret division of a unit fraction by a non-zero whole number and compute the quotient.
**5.NF.F.7B** Interpret division of a whole number by a unit fraction and compute the quotient.
**5.NF.F.7C** Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions by using visual fraction models and equations to represent the problem.

In addition to Proficient, the **Advanced** student is able to:

- Create and solve a real-world problem and justify how the model/equation relates to the problem. **OR**
- Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers using multiple representations.

The **Proficient** student is able to extend the concept of division to divide unit fractions and whole numbers by using visual fraction models and equations.
A. Interpret division of a unit fraction by a non-zero whole number and compute the quotient.
B. Interpret division of a whole number by a unit fraction and compute the quotient.
C. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions by using visual fraction models and equations to represent the problem.

The **Basic** student is able to:
A. Interpret division of a unit fraction by a non-zero whole number and compute the quotient when given a visual model.
B. Interpret division of a whole number by a unit fraction and compute the quotient when given a visual model.

The **Below Basic** student may be able to:
A. Interpret division of a unit fraction by a non-zero whole number and compute the quotient with partial success when given a visual model.
B. Interpret division of a whole number by a unit fraction and compute the quotient with partial success when given a visual model.

# MEASUREMENT AND DATA

## Convert like measurement units within a given measurement system.

**5.MD.G.1 Understand a coordinate system.**

In addition to Proficient, the **Advanced** student is able to:

- Convert different-sized standard measurement units, within a given measurement system, requiring multiple conversions. Solve real-world problems with multiple steps involving these conversions. **OR**
- Create and solve multi-step real-world problems by converting different-sized standard measurement units within a given measurement system. **OR**
- Convert different-sized standard measurement units within multiple measurement systems to solve real-world problems.

The **Proficient** student is able to solve multi-step real-world problems by converting among different-sized standard measurement units within a given measurement system.

The **Basic** student is able to solve multi-step real-world problems by converting among different-sized standard measurement units, within a given measurement system, when given the conversion equivalence.

The **Below Basic** student may be able to solve multi-step real-world problems by converting among different-sized standard measurement units, within a given measurement system, with partial success, when given the conversion equivalence.

## Represent and interpret data.

**5.MD.H.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions to solve problems involving information presented in line plots.**

In addition to Proficient, the **Advanced** student is able to:

- Predict ways in which operations with fractions, and fractions on a line plot, would change if the data set were changed. **AND**
- Use three or more operations with fractions to solve problems involving information presented in line plots.

The **Proficient** student is able to make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions to solve problems involving information presented in line plots.

The **Basic** student is able to:

- Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). **OR**
- Use operations on fractions to solve problems involving information presented in line plots.

The **Below Basic** student may be able to:

- Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$) with partial success. **OR**
- Use operations on fractions to solve problems involving information presented in line plots with partial success.

## Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

**5.MD.I.3 Recognize volume as an attribute of three-dimensional figures and understand concepts of volume measurement such as "unit cube" and a volume of n cubic units.**

In addition to Proficient, the **Advanced** student is able to explain how to find missing dimension(s) when given volume and justify.

The **Proficient** student is able to recognize volume as an attribute of three-dimensional figures and understand concepts of volume measurement such as "unit cube" and a volume of n cubic units.

The **Basic** student is able to identify the use of volume and the appropriate measurement for a given situation or model.

The **Below Basic** student may be able to identify the use of volume and the appropriate measurement for a given situation or model with partial success.

**5.MD.I.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft., and improvised units.**

In addition to Proficient, the **Advanced** student is able to:

- Use an efficient counting strategy (not counting one unit at a time) and determine appropriate measurement units. **OR**
- Explain how you would find volume in a compound figure.

The **Proficient** student is able to measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft., and improvised units.

The **Basic** student is able to measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units when given a partially completed visual model.

The **Below Basic** student may be able to measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units, with partial success, when given a partially completed visual model.

**5.MD.I.5 Relate volume to the operations of multiplication and solve real-world and mathematical problems involving volume.**

**5.MD.I.5A** Find the volume of a right rectangular prism with whole number dimensions by multiplying them. Show that this volume is the same as when counting unit cubes.

**5.MD.I.5B** Find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems given the formulas $V = (l)(w)(h)$ and $V = (B)(h)$ for rectangular prisms.

In addition to Proficient, the **Advanced** student is able to:

- Solve real-world problems by finding volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts. **OR**
- Find the base or height when given volume and some of the dimensions. **OR**
- Find multiple possible dimensions of a right rectangular prism with a given volume.

The **Proficient** student is able to relate volume to the operations of multiplication and solve real-world and mathematical problems involving volume.

A. Find the volume of a right rectangular prism with whole number dimensions by multiplying them. Show that this volume is the same as when counting unit cubes.

B. Find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems given the formulas $V = (l)(w)(h)$ and $V = (B)(h)$ for rectangular prisms.

The **Basic** student is able to:

A. Find the volume of a right rectangular prism with whole number dimensions by multiplying them. Show that this volume is the same as when counting unit cubes. **OR**

B. Find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems given the formulas $V = (l)(w)(h)$ and $V = (B)(h)$ for rectangular prisms.

The **Below Basic** student may be able to:

A. Find the volume of a right rectangular prism with whole number dimensions by multiplying them with partial success. **OR**

B. Find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems given the formulas $V = (l)(w)(h)$ and $V = (B)(h)$ for rectangular prisms with partial success.

## GEOMETRY

### Graph points on the coordinate plane to solve real-world and mathematical problems.

**5.G.J.1 Understand a coordinate system.**

**5.G.J.1A** The x- and y- axes are perpendicular number lines that intersect at 0 (the origin).

**5.G.J.1B** Any point on the coordinate plane can be represented by its coordinates.

**5.G.J.1C** The first number in an ordered pair is the x-coordinate and represents the horizontal distance from the origin.

**5.G.J.1D** The second number in an ordered pair is the y-coordinate and represents the vertical distance from the origin.

In addition to Proficient, the **Advanced** student is able to show understanding of the coordinate system in real-world situations (first quadrant only).

The **Proficient** student is able to understand a coordinate system.

A. The x- and y- axes are perpendicular number lines that intersect at 0 (the origin).

B. Any point on the coordinate plane can be represented by its coordinates.

C. The first number in an ordered pair is the x-coordinate and represents the horizontal distance from the origin.

D. The second number in an ordered pair is the y-coordinate and represents the vertical distance from the origin.

The **Basic** student is able to identify the components of a coordinate system.

A. The x- and y- axes are perpendicular number lines that intersect at 0 (the origin).

B. Any point on the coordinate plane can be represented by its coordinates.

C. The first number in an ordered pair is the x-coordinate and represents the horizontal distance from the origin.

D. The second number in an ordered pair is the y-coordinate and represents the vertical distance from the origin.

The **Below Basic** student may be able to identify the components of a coordinate system with partial success.

A. The x- and y- axes are perpendicular number lines that intersect at 0 (the origin).

B. Any point on the coordinate plane can be represented by its coordinates.

C. The first number in an ordered pair is the x-coordinate and represents the horizontal distance from the origin.

D. The second number in an ordered pair is the y-coordinate and represents the vertical distance from the origin.

**5.G.J.2 Plot and interpret points in the first quadrant of the coordinate plane to represent real-world and mathematical situations.**

In addition to Proficient, the **Advanced** student is able to describe the x- and y-coordinate's position when mathematical operations are performed on the coordinates in any of the four quadrants.

The **Proficient** student is able to plot and interpret points in the first quadrant of the coordinate plane to represent real-world and mathematical situations.

The **Basic** student is able to locate a point in the first quadrant using an ordered pair.

The **Below Basic** student may be able to locate a point in the first quadrant using an ordered pair with partial success.

## Classify two-dimensional figures into categories based on their properties.

**5.G.K.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category**.

**Assessment Boundary:** Use polygons only.

In addition to Proficient, the **Advanced** student is able to formulate logical arguments to show that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

**Assessment Boundary:** Use polygons only.

The **Proficient** student is able to understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

**Assessment Boundary:** Use polygons only.

The **Basic** student is able to classify two-dimensional figures into basic subcategories.

**Assessment Boundary:** Use polygons only.

The **Below Basic** student may be able to classify two-dimensional figures into basic subcategories with partial success.

**Assessment Boundary:** Use polygons only.

**5.G.K.4 Classify polygons in a hierarchy based on properties.**

In addition to Proficient, the **Advanced** student is able to construct polygons according to given attributes.

The **Proficient** student is able to classify polygons in a hierarchy based on properties.

The **Basic** student is able to:

- Classify two-dimensional figures into two basic subcategories. **OR**
- Identify specific properties of the subcategories.

The **Below Basic** student may be able to:

- Classify two-dimensional figures into two basic subcategories with partial success. **OR**
- Identify specific properties of the subcategories with partial success.

**Companion document to the 2018 Mathematics Content Standards**

# Grade 6 Mathematics Content Standards & PLDs

## GRADE 6 MATH PRACTICES

### MP1 Make sense of problems and persevere in solving them.

**6.MP.1** In grade 6, students solve problems involving ratios and rates and discuss (verbally or in writing) how they solve them. Students analyze the problem (including what is given, not given, and what is being asked), identify what strategies are needed, recognize multiple pathways to a solution, and make an initial attempt to solve the problem. Students analyze the result for validity and refine strategies if necessary.

### MP2 Reason abstractly and quantitatively.

**6.MP.2** Students recognize a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students begin to contextualize to understand the meaning of the number or variable as it relates to the problem.

### MP3 Construct viable arguments and critique the reasoning of others.

**6.MP.3** Students begin to contextualize to understand the meaning of the number or variable as it relates to the problem. They make conjectures, explore validity, reason mathematically, justify, evaluate their own thinking.

### MP4 Model with mathematics.

**6.MP.4** Students can clearly show their work by using diagrams, words, symbols or pictures. They are able to identify important quantities in a practical situation and map their relationships using tools such as, diagrams, two-way tables, graphs, flowcharts or formulas. They can recognize and analyze those relationships mathematically to draw conclusions. They can interpret their mathematical results of problems involving non-negative rational numbers in the context of the situation and reflect on whether the results make sense.

### MP5 Use appropriate tools strategically.

**6.MP.5** Students consider available tools (including estimation, concrete models, and technology), and decide when certain tools might be helpful. They choose the representation (table, graph, equation, words) that best suits the problem. Students use concrete models to develop insight into ratios and other concepts. Students extend this insight to more abstract representations, including pictures and symbols. Students understand the limitations of each tool. Tools might include: unifix cubes, fraction bars, base-ten blocks, number lines, graph paper, calculator, paper and pencil, and others.

### MP6 Attend to precision.

**6.MP.6** Students continue to refine their mathematical communication and reasoning skills by using clear language in their discussions with others. Students define variables, including their relationship, specify units of measure, and label each axis accurately. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities. Students use appropriate symbols, labels, and units of measure when solving problems with calculations that are accurate and efficient. The answer to the problem matches what was asked in the problem.

### MP7 Look for and make use of structure.

**6.MP.7** Students routinely seek patterns or structure to model and solve problems. They recognize that patterns exist in ratio tables. Students notice patterns and identify strategies for creating equivalent expressions. Students identify complicated expressions or figures as compositions of simple parts.

### MP8 Look for and express regularity in repeated reasoning.

**6.MP.8** Students use repeated reasoning to understand algorithms and make generalizations about patterns. They construct examples and models that confirm their generalization. They develop short cuts and check for reasonableness of answers. Students ask questions such as, "How would we verify that?" and "How is this similar to patterns with whole numbers?"

## RATIOS AND PROPORTIONAL RELATIONSHIPS

### Understand ratio concepts and use ratio reasoning to solve problems.

**6.RP.A.1 Students use repeated reasoning to understand algorithms and make generalizations about patterns. They construct examples and models that confirm their generalization. They develop short cuts and check for reasonableness of answers. Students ask questions such as, "How would we verify that?" and "How is this similar to patterns with whole numbers?"**

In addition to Proficient, the **Advanced** student is able to manipulate and make connections between different representations for ratio relationships.
The **Proficient** student is able to interpret a ratio relationship between two quantities, including part-to-part and part-to-whole.
The **Basic** student is able to write a ratio relationship between two quantities.
The **Below Basic** student may be able to identify a ratio using a mathematical or verbal representation.

**6.RP.A.2 Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.**

In addition to Proficient, the **Advanced** student is able to interpret a unit rate from a visual representation using unit rate language.
**Assessment Boundary:** Do not use complex fractions or negatives.
The **Proficient** student is able to write a unit rate to compare two quantities using rational numbers and use unit rate language to describe two quantities in the context of a ratio relationship.
**Assessment Boundary:** Do not use complex fractions or negatives.
The **Basic** student is able to write a unit rate to compare two quantities using whole numbers.
**Assessment Boundary:** Do not use complex fractions or negatives.
The **Below Basic** student may be able to identify unit rates.
**Assessment Boundary:** Do not use complex fractions or negatives.

**6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems.**

**6.RP.A.3A** Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

**6.RP.A.3B** Solve unit rate problems including those involving unit pricing and constant speed.

**6.RP.A.3C** Understand that a percentage is a rate per 100 and use this to solve problems involving wholes, parts, and percentages.

**6.RP.A.3D** Use ratio reasoning to convert measurement units; convert units appropriately when multiplying or dividing quantities.

In addition to Proficient, the **Advanced** student is able to:

A. Extend ratio and rate reasoning beyond what is displayed in a table or graph.
B. Solve unit rate problems that require determining a unit rate with a positive rational numerator and whole number denominator.
C. In mathematical and real-world contexts solve two-step problems involving wholes, parts, and percentages.
D. Use ratio reasoning to convert measurement units and transform units appropriately when multiplying and dividing in two-step problems.

The **Proficient** student is able to:

A. Make tables of equivalent ratios relating quantities with whole number measurements and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
B. Solve unit rate problems with whole number measurements including those involving unit pricing and constant speed.
C. In mathematical and real-world contexts solve one-step problems involving wholes, parts, and percentages.
D. Use ratio reasoning to convert measurement units and to transform units appropriately when multiplying or dividing quantities in one-step problems.

The **Basic** student is able to:

A. Make a table of equivalent ratios relating quantities with whole number measurements.
B. Solve unit rate problems given the unit rate with whole number measurements.
C. Solve one-step problems involving wholes, parts, or percentages.
D. Use ratio reasoning in a one-step problem to convert measurement units within the same system.

The **Below Basic** student may be able to:

A. Find missing values in a table.
B. Identify unit rate with whole number measurements.
C. Identify the percent of a quantity as a rate per hundred.
D. Identify ratio relationships using measurement units within the same system.

## THE NUMBER SYSTEM

### Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

**6.NS.B.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions by using visual fraction models and equations to represent the problem.**

In addition to Proficient, the **Advanced** student is able to interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions by using visual fraction models and equations to represent the problem in a real-world context.

The **Proficient** student is able to interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions by using visual fraction models and equations to represent the problem.

The **Basic** student is able to compute quotients of a fraction by a unit fraction. Students solve problems in mathematical contexts involving division of a fraction by a unit fraction.
The **Below Basic** student may be able to solve problems in mathematical contexts involving division of a whole number by a unit fraction.

**6.NS.B.2 Divide multi-digit numbers using efficient and generalizable procedures including, but not limited to the standard algorithm.**

**Assessment Boundary:** Use up to 5-digit dividends, 2-digit divisors.
In addition to Proficient, the **Advanced** student is able to divide multi-digit numbers with fractional remainders using efficient and generalizable procedures including, but not limited to the standard algorithm and explain the reasonableness of the result.
**Assessment Boundary:** Use up to 5-digit dividends, 2-digit divisors.
The **Proficient** student is able to divide multi-digit numbers using efficient and generalizable procedures including, but not limited to the standard algorithm.
**Assessment Boundary:** Use up to 5-digit dividends, 2-digit divisors.
The **Basic** student is able to divide three-digit or four-digit dividends by two-digit divisors resulting in no remainder using efficient and generalizable procedures including, but not limited to the standard algorithm.
**Assessment Boundary:** Use up to 5-digit dividends, 2-digit divisors.
The **Below Basic** student may be able to divide two-digit dividends by one-digit divisors resulting in no remainder using efficient and generalizable procedures, including, but not limited to the standard algorithm.
**Assessment Boundary:** Use up to 5-digit dividends, 2-digit divisors.

## Compute fluently with multi-digit numbers and find common factors and multiples.

**6.NS.C.3 Add, subtract, multiply, and divide manageable multi-digit decimals using efficient and generalizable procedures including, but not limited to the standard algorithm for each operation.**

In addition to Proficient, the **Advanced** student is able to add, subtract, multiply, and divide multi-digit decimals using efficient and generalizable procedures including, but not limited to the standard algorithm for each operation and explain the reasonableness of the answer.
**Assessment Boundary:** Limit decimals in the given values to the hundredths place.
The **Proficient** student is able to add, subtract, multiply, and divide multi-digit decimals using efficient and generalizable procedures including, but not limited to the standard algorithm for each operation.
**Assessment Boundary:** Limit decimals in the given values to the hundredths place.
The **Basic** student is able to add, subtract, multiply, and divide decimals to tenths using efficient and generalizable procedures including, but not limited to the standard algorithm for each operation.
The **Below Basic** student may be able to add, subtract, multiply, **or** divide decimals to tenths using efficient and generalizable procedures including, but not limited to the standard algorithm for each operation.

**6.NS.C.4 Find common factors and multiples using two whole numbers.**

**6.NS.C.4A** Find the greatest common factor of two whole numbers less than or equal to 100.

**6.NS.C.4B** Find the least common multiple of two whole numbers less than or equal to 12.

**6.NS.C.4C** Use the Distributive Property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

In addition to Proficient, the **Advanced** student is able to find common factors and multiples using two whole numbers.

A. Find the greatest common factor of multiple whole numbers less than or equal to 100.
B. Find the least common multiple of three whole numbers less than or equal to 12.
C. Explain why two given expressions written in factored and distributed form are equivalent using appropriate mathematical language.

The **Proficient** student is able to find common factors and multiples using two whole numbers.

A. Find the greatest common factor of two whole numbers less than or equal to 100.
B. Find the least common multiple of two whole numbers less than or equal to 12.
C. Use the Distributive Property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

The **Basic** student is able to find common factors and multiples using two whole numbers.

A. Find the greatest common factor of two whole numbers less than or equal to 24.
B. Find the least common multiple of two prime numbers less than or equal to 12.
C. Use the Distributive Property to express a sum of two whole numbers 1–24 with a common factor as a multiple of a sum of two whole numbers with no common factor.

The **Below Basic** student may be able to find common factors and multiples using two whole numbers.

A. Given a visual model, identify the greatest common factor of two whole numbers less than or equal to 24.
B. Given multiples, identify the least common multiple of two whole numbers less than or equal to 12.
C. Given the common factors, use the Distributive Property to express a sum of two whole numbers 1–24 with a common factor as a multiple of a sum of two whole numbers with no common factor.

## Apply and extend previous understandings of numbers to the system of rational numbers.

**6.NS.D.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values and use them to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.**

In addition to Proficient, the **Advanced** student is able to recognizes patterns and makes generalizations about characteristics of positive and negative numbers in real-world contexts (may use any rational number, including fractions and decimals).

The **Proficient** student is able to understand that positive and negative integers are used together to describe quantities having opposite directions or values and use them to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

The **Basic** student is able to place integers on the number line. In a given situation (e.g., elevation, sea level), student is able to determine the meaning of 0.

The **Below Basic** student may be able to place integers on the number line (with whole-number increments), extending the counting pattern to integers.

**6.NS.D.6 Extend the understanding of the number line to include all rational numbers and apply this concept to the coordinate plane.**

**6.NS.D.6A** Understand the concept of opposite numbers, including 0, and their relative locations on the number line.

**6.NS.D.6B** Understand that signs of numbers in ordered pairs indicate locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

**6.NS.D.6C** Find and position rational numbers on a horizontal or vertical number line diagram; find and position pairs of rational numbers on a coordinate plane.

In addition to Proficient, the **Advanced** student is able to:

A. Create a real-world situation to demonstrate opposite numbers.

B. Given one ordered pair, create a reflection across the x- and y-axis.

C. Find and position rational numbers on a horizontal or vertical number line diagram when the scale is not given; find and position pairs of rational numbers on a coordinate plane when the scale is not given.

The **Proficient** student is able to extend the understanding of the number line to include all rational numbers and apply this concept to the coordinate plane.

A. Represent the concept of opposite numbers, including 0, and their relative locations on the number line in real-world contexts.

B. Given ordered pairs that differ by a sign with x-coordinate and/or y-coordinate, recognize that the location of the points are related by reflections across one or both axes.

C. Find and position rational numbers on a horizontal or vertical number line diagram when the scale is given; find and position pairs of rational numbers on a coordinate plane when the scale is given.

The **Basic** student is able to:

A. Place two opposite numbers on a number line in a mathematical context.

B. Identify/determine the quadrant from a given coordinate.

C. Graph integer values on a horizontal or vertical number line and graph ordered pairs of integers in all four quadrants of a coordinate plane.

The **Below Basic** student may be able to:

A. Place the opposite number on a number line given one.

B-C. Plot whole-number ordered pairs in the first quadrant of a coordinate plane (with one-unit increments on both axes).

**6.NS.D.7 Understand ordering and absolute value of rational numbers.**

**6.NS.D.7A** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.

**6.NS.D.7B** Write, interpret, and explain statements of order for rational numbers in real-world contexts.

**6.NS.D.7C** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.

**6.NS.D.7D** Distinguish comparisons of absolute value from statements about order.

In addition to Proficient, the **Advanced** student is able to apply ordering and absolute value of rational numbers (including percent form).

A. Create statements of inequality about the relative position of two numbers on a (vertical or horizontal) number line diagram.

B. Write, interpret, and explain statements of order for rational numbers in real-world contexts.

C. Describe the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.

D. Distinguish comparisons of absolute value from statements about order.

The **Proficient** student is able to understand ordering and absolute value of rational numbers.

A. Interpret statements of inequality as statements about the relative position of two numbers on a (vertical or horizontal) number line diagram.
B. Write, interpret, and explain statements of order for rational numbers in real-world contexts.
C. Describe the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
D. Distinguish comparisons of absolute value from statements about order.

The **Basic** student is able to understand ordering and absolute value of integers.
A. Interpret statements of inequality as statements about the relative position of two numbers on a (vertical or horizontal) number line diagram.
B. Write, interpret, and explain statements of order for integers in real-world contexts.
C. Describe the absolute value of an integer as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
D. Distinguish comparisons of absolute value from statements about order.

The **Below Basic** student may be able to understand ordering and absolute value of whole numbers.
A. Interpret statements of inequality as statements about the relative position of two numbers on a (vertical or horizontal) number line diagram.
B. Write, interpret, and explain statements of order for whole numbers in real-world contexts.
C. Describe the absolute value of a whole number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive quantity in a real-world situation.
D. Distinguish comparisons of absolute value from statements about order.

**6.NS.D.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Find distances between points with the same first coordinate or the same second coordinate; relate absolute value and distance.**

In addition to Proficient, the **Advanced** student is able to solve real-world and mathematical problems by graphing non-integer points in all four quadrants of the coordinate plane. Find distances between non-integer points with the same first coordinate or the same second coordinate; relate absolute value and distance.
The **Proficient** student is able to solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Find distances between points with the same first coordinate or the same second coordinate; relate absolute value and distance.
The **Basic** student is able to solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Find distances between points with the same first coordinate or the same second coordinate in the same quadrant.
The **Below Basic** student may be able to solve real-world and mathematical problems by graphing points in quadrant one of the coordinate plane. Find distances between points with the same first coordinate or the same second coordinate.

## EXPRESSIONS AND EQUATIONS

### Apply and extend previous understandings of arithmetic to algebraic expressions.

**6.EE.E.1 Write and evaluate numerical expressions involving whole-number exponents.**

In addition to Proficient, the **Advanced** student is able to write and evaluate numerical multi-step expressions involving the Distributive Property and whole number exponents.
The **Proficient** student is able to write and evaluate multi-step numerical expressions involving whole number exponents.
The **Basic** student is able to write and evaluate two-step numerical expressions involving one whole number exponent.
The **Below Basic** student may be able to write and evaluate one-step numerical expressions involving one whole number exponent.

**6.EE.E.2 Write, read, and evaluate expressions in which letters stand for numbers.**

**6.EE.E.2A** Write expressions that record operations with numbers and with letters standing for numbers.

**6.EE.E.2B** Identify parts of an expression using mathematical terms (sum, difference, term, product, factor, quotient, coefficient, constant).

**6.EE.E.2C** Use Order of Operations to evaluate algebraic expressions using positive rational numbers and whole-number exponents. Include expressions that arise from formulas in real-world problems.

In addition to Proficient, the **Advanced** student is able to write, read, and evaluate expressions in which letters stand for numbers.

A. Write algebraic expressions using grouping symbols.
B. Create an expression given mathematical terms (sum, difference, term, product, factor, quotient, coefficient, constant).
C. Use Order of Operations to justify the evaluation of algebraic expressions that contain positive rational numbers and whole-number exponents. Include expressions that arise from formulas relative to sixth grade standards in real-world problems.

The **Proficient** student is able to write, read, and evaluate expressions in which letters stand for numbers.

A. Write two-step algebraic expressions.
B. Identify parts of an expression using mathematical terms (sum, difference, term, product, factor, quotient, coefficient, constant).
C. Use Order of Operations to evaluate algebraic expressions using positive rational numbers and whole-number exponents. Include expressions that arise from formulas relative to sixth grade standards in real-world problems.

The **Basic** student is able to write, read, and evaluate expressions in which letters stand for numbers.

A. Write one-step algebraic expressions.
B. Identify a part of an expression using mathematical terms (sum, difference, term, product, factor, quotient, coefficient, constant).
C. Use Order of Operations to evaluate algebraic expressions using whole numbers and whole-number exponents. Include expressions that arise from formulas relative to sixth grade geometry standards in real-world problems.

The **Below Basic** student may be able to write, read, and evaluate expressions in which letters stand for numbers.

A. Identify an algebraic expression.
B. Identify the number of terms in a mathematical expression.
C. Use Order of Operations to evaluate algebraic expressions using whole numbers.

**6.EE.E.3 Apply the properties of operations to generate equivalent expressions.**

In addition to Proficient, the **Advanced** student is able to justify why two expressions are equivalent using the properties of operations.

The **Proficient** student is able to apply the properties of operations to generate equivalent expressions (Commutative Property, Associative Property, Distributive Property, Additive Identity Property, Multiplicative Identity Property, and Zero Product Property).

The **Basic** student is able to identify the properties of operations in equivalent expressions (Commutative Property, Associative Property, Distributive Property, Additive Identity Property, Multiplicative Identity Property, and Zero Product Property).

The **Below Basic** student may be able to identify the properties of operations in equivalent expressions (Commutative Property, Associative Property, Distributive Property).

**6.EE.E.4 Identify when two expressions are equivalent.**

In addition to Proficient, the **Advanced** student is able to generate equivalent expressions.

The **Proficient** student is able to identify when two expressions are equivalent with positive rational numbers.

The **Basic** student is able to identify a model that matches an expression.

The **Below Basic** student may be able to identify when two numerical expressions are equivalent.

## Reason about and solve one-variable equations and inequalities.

**6.EE.F.5 Understand a solution to an equation or an inequality makes the equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.**

In addition to Proficient, the **Advanced** student is able to use substitution to determine whether a given rational number in a specified set makes a two-step equation or a two-step inequality true.
The **Proficient** student is able to use substitution to determine whether a given non-negative rational number in a specified set makes a one-step equation or a one-step inequality true.
The **Basic** student is able to use substitution to determine whether a given whole number in a specified set makes a one-step equation or a one-step inequality true.
The **Below Basic** student may be able to use substitution to determine whether a given whole number in a specified set makes a one-step equation true.

**6.EE.F.6 Use variables to represent unknown numbers and write expressions when solving a real-world or mathematical problem.**

In addition to Proficient, the **Advanced** student is able to use variables to represent unknown numbers and write multiple expressions to represent the same real-world or mathematical problem and justify that they are equivalent.
The **Proficient** student is able to use variables to represent unknown numbers and write expressions to represent real-world or mathematical problems.
The **Basic** student is able to use variables to represent unknown numbers and write one-step expressions to represent real-world or mathematical problems.
The **Below Basic** student may be able to identify a variable representing an unknown number.

**6.EE.F.7 Write and solve real-world and mathematical problems in the form of one-step, linear equations involving nonnegative rational numbers.**

In addition to Proficient, the **Advanced** student is able to solve problems in both real-world and mathematical contexts by writing and solving equations in the form of two-step, linear equations involving nonnegative rational numbers.
The **Proficient** student is able to solve problems in both real-world and mathematical contexts by writing and solving equations in the form of one-step, linear equations involving nonnegative rational numbers.
The **Basic** student is able to solve problems in both real-world and mathematical contexts by writing and solving equations in the form of one-step, linear equations involving whole numbers.
The **Below Basic** student may be able to solve equations in the form of one-step, linear equations involving whole numbers.

**6.EE.F.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.**

In addition to Proficient, the **Advanced** student is able to write an inequality of the form $x \geq c$ or $x \leq c$ (with the variable on either side) to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities have infinitely many solutions and represent solutions by graphing on a number line.
The **Proficient** student is able to write an inequality of the form $x > c$ or $x < c$ (with the variable on either side) to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities have infinitely many solutions and represent solutions by graphing on a number line.
The **Basic** student is able to write an inequality of the form $x > c$ or $x < c$ (with the variable on the left) to represent a constraint or condition in a real-world or mathematical problem. Represent solutions by graphing on a number line.
The **Below Basic** student may be able to recognize the value on an inequality of the form $x > c$ or $x < c$ (with the variable on the left) on number line diagrams.

### Represent and analyze quantitative relationships between dependent and independent variables.

**6.EE.G.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity (dependent variable), in terms of the other quantity (independent variable). Analyze their relationship using graphs and tables, and relate these to the equation.**

In addition to Proficient, the **Advanced** student is able to create a real-world problem from a given table, graph, or equation and justify the relationship between the dependent and independent variables.
The **Proficient** student is able to use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity (dependent variable) in terms of the other quantity (independent variable). Analyze their relationship using graphs and tables, and relate these to the equation.
The **Basic** student is able to describe the relationship between dependent and independent variables from a table or graph.
The **Below Basic** student may be able to complete a table from a given equation.

## GEOMETRY

### Solve real-world and mathematical problems involving area, surface area, and volume.

**6.G.H.1 Find area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.**

In addition to Proficient, the **Advanced** student is able to find a missing dimension given the area of a triangle or a special quadrilateral and all but one dimension.
The **Proficient** student is able to find area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
The **Basic** student is able to find area of right triangles, other triangles, squares, and rectangles using their respective formulas or strategies.
The **Below Basic** student may be able to decompose a polygon into right triangles and rectangles.

**6.G.H.2 Find the volume of a right rectangular prism with fractional edge lengths in the context of solving real-world and mathematical problems by applying the formulas V = (l)(w)(h) and V = (B)(h), and label with appropriate units.**

In addition to Proficient, the **Advanced** student is able to find a missing dimension when given the volume of a right rectangular prism with at least one fractional edge length.
The **Proficient** student is able to find the volume of a right rectangular prism with fractional edge lengths in the context of solving real-world and mathematical problems by applying the formulas $V = (l)(w)(h)$ and $V = (B)(h)$, and label with appropriate units.
The **Basic** student is able to find the volume of a right rectangular prism with **one** fractional edge length in the context of solving real-world and mathematical problems by applying the formulas $V = (l)(w)(h)$ and $V = (B)(h)$, and label with appropriate units.
The **Below Basic** student may be able to find the volume of a right rectangular prism with whole number edge lengths.

**6.G.H.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.**

In addition to Proficient, the **Advanced** student is able to create and graph a polygon with a given perimeter, area, or dimensions on a coordinate plane; justify the solution by listing ordered pairs and dimensions of the polygon.

The **Proficient** student is able to draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems including perimeter and area.
The **Basic** student is able to draw a right triangle or a rectangle in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of mathematical problems involving area.
The **Below Basic** student may be able to use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate when given a rectangle in the coordinate plane with coordinates for the vertices. Apply these techniques in the context of mathematical problems involving area.

**6.G.H.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures in the context of solving real-world and mathematical problems.**

In addition to Proficient, the **Advanced** student is able to:

- Create nets with different dimensions that represent right rectangular prisms with the same surface area. **OR**
- Compare nets with a fixed volume to determine maximum or minimum surface area.

The **Proficient** student is able to represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of right triangular prisms, right rectangular prisms, and right rectangular pyramids (given lateral height) in the context of solving real-world and mathematical problems.
The **Basic** student is able to represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of right rectangular prisms.
The **Below Basic** student may be able to identify the net of a right prism.

## STATISTICS AND PROBABILITY

### Develop understanding of statistical variability.

**6.SP.I.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.**

In addition to Proficient, the **Advanced** student is able to create a statistical question and explain the variability in the answer.
The **Proficient** student is able to recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
The **Basic** student is able to change a question from a nonstatistical question to a statistical question.
The **Below Basic** student may be able to recognize a statistical question from a list of questions.

**6.SP.I.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.**

In addition to Proficient, the **Advanced** student is able to make generalizations about a numerical data set collected to answer a statistical question that has a distribution which can be described by its center (median, mode, or mean), spread (range or interquartile range), and overall shape (symmetry).
The **Proficient** student is able to show that a visual representation of a set of data collected to answer a statistical question has a distribution which can be described by its center (median, mode, or mean), spread (range or interquartile range), and overall shape (symmetry).

The **Basic** student is able to show that a visual representation of a set of data collected to answer a statistical question has a distribution which can be described by its spread (range or interquartile range) and overall shape (symmetry).
The **Below Basic** student may be able to show that a visual representation of a set of data collected to answer a statistical question has a distribution which can be described by its overall shape (symmetry).

**6.SP.I.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.**

In addition to Proficient, the **Advanced** student is able to:

- Create questions that could be answered from the measures of center or the measures of variation. **OR**
- Determine the kinds of questions that can be answered using the measures of center and the measures of variation.

The **Proficient** student is able to compare and contrast the measures of center for a numerical data set that summarizes all of its values with a single number and the measures of variation that describe how its values vary with a single number.
The **Basic** student is able to compare and contrast the measures of center (mean or median) for a numerical data set that summarizes all of its values with a single number and the measure of variation (range) that describes how its values vary with a single number.
The **Below Basic** student may be able to identify a measure of center (mean or median) for a numerical data set that summarizes all of its values with a single number.

## Summarize and describe distributions.

**6.SP.J.4 Display numerical data in plots on a number line, including dot plots, stem-and-leaf plots, histograms, and box plots.**

In addition to Proficient, the **Advanced** student is able to determine the appropriate display to represent numerical data (e.g., plot on a number line, including dot plots or histograms) and justify the choice.
The **Proficient** student is able to display numerical data in plots on a number line, including dot plots, stem-and-leaf plots, histograms, and box plots.
The **Basic** student is able to identify from a given data set a corresponding representation including dot plots, stem-and-leaf plots, histograms, and box plots.
The **Below Basic** student may be able to match a visual representation of data to the name of the graph (dot plot, stem-and-leaf plot, histogram, or box plot).

**6.SP.J.5 Summarize numerical data sets in relation to their real-world context.**

**6.SP.J.5A** Report the sample size.
**6.SP.J.5B** Describe the context of the data under investigation, including how it was measured and its units of measurement.
**6.SP.J.5C** Find quantitative measures of center (median, mode and mean) and variability (range and interquartile range). Describe any overall pattern (including outliers, clusters, and distribution), with reference to the context in which the data was gathered.
**6.SP.J.5D** Justify the choice of measures of center (median, mode, or mean) based on the shape of the data distribution and the context in which the data was gathered.

In addition to Proficient, the **Advanced** student is able to compare and make generalizations about two different sets of numerical data in relation to their real-world context including:

A. Report sample size.
B. Describe the context of the data under investigation, including how it was measured and its units of measurement.

C. Find quantitative measures of center (median, mode, and mean) and variability (range and interquartile range). Describe any overall pattern (including outliers, clusters, and distribution), with reference to the context in which the data was gathered.
D. Justify the choice of measures of center (median, mode, or mean) based on the shape of the data distribution and the context in which the data was gathered.

The **Proficient** student is able to summarize numerical data sets in relation to their real-world context.

A. Report the sample size.
B. Describe the context of the data under investigation, including how it was measured and its units of measurement.
C. Find quantitative measures of center (median, mode, and mean) and variability (range and interquartile range). Describe any overall pattern (including outliers, clusters, and distribution), with reference to the context in which the data was gathered.
D. Justify the choice of measures of center (median, mode, or mean) based on the shape of the data distribution and the context in which the data was gathered.

The **Basic** student is able to summarize numerical data sets in relation to their real-world context.

A. Report sample size.
B. Describe the context of the data under investigation, including how it was measured and its units of measurement.
C. Find quantitative measures of center (median, mode, and mean) and variability (range and interquartile range).

The **Below Basic** student may be able to summarize numerical data sets in relation to their real-world context by finding quantitative measures of center (median, mode, and mean) and variability (range).

**Companion document to the 2018 Mathematics Content Standards**

# Grade 7 Mathematics Content Standards & PLDs

## GRADE 7 MATH PRACTICES

### MP1 Make sense of problems and persevere in solving them.

**7.MP.1** In grade 7, students solve real-world problems involving ratios, rates, proportions, rational numbers and geometric concepts and discuss (verbally or in writing) how they solve them. Students analyze the problem (including what is given, not given, and what is being asked), identify what strategies are needed, choose an appropriate pathway, then make an initial attempt to solve the problem. Students analyze the result for validity and refine strategies if necessary.

### MP2 Reason abstractly and quantitatively.

**7.MP.2** Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

### MP3 Construct viable arguments and critique the reasoning of others.

**7.MP.3** Students construct arguments using verbal or written explanations that involve solving problems with rational numbers. They make conjectures, explore validity, reason mathematically, justify, evaluate their own thinking and the thinking of other students.

### MP4 Model with mathematics.

**7.MP.4** Students can clearly show their work by using diagrams, words, symbols or pictures. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and/or formulas. They can analyze those relationships mathematically to draw conclusions. They interpret their mathematical results of problems involving rational numbers in the context of the situation and reflect on whether the results make sense.

### MP5 Use appropriate tools strategically.

**7.MP.5** Students consider available tools (including estimation, concrete models, and technology as appropriate), and decide when certain tools might be helpful. Students develop more efficacy with technology. They choose the representation (table, graph, equation, words) that best suits the problem. Students use concrete models to develop insight into proportions and other concepts. Students then extend this insight to more abstract representations, including pictures and symbols. Students understand the limitations of each tool. Tools might include: integer tiles, algebra tiles, geometric nets, number lines, graphing technology, scientific calculator, paper and pencil, and others.

### MP6 Attend to precision.

**7.MP.6** Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, including their relationship, specify units of measure, and label each axis accurately. Student use appropriate terminology when referring to rates, ratios, proportions, probability models, geometric figures, data displays, and components of expressions, equations or inequalities.

Students use appropriate symbols, labels, and units of measure when solving problems with calculations that are accurate and efficient. Answer to the problem matches what was asked in the problem.

### MP7 Look for and make use of structure.

**7.MP.7** Students routinely seek patterns or structure to model and solve problems. They recognize that patterns exist in ratio tables and make connections with the constant of proportionality in a table and the slope of a graph. Students recognize patterns and identify and develop strategies for creating equivalent expressions. Students identify complicated expressions or figures as compositions of simple parts.

### MP8 Look for and express regularity in repeated reasoning.

**7.MP.8** Students routinely seek patterns or structure to model and solve problems. They apply properties to solve problems based upon patterns they have identified. Students examine patterns to generate equations and describe relationships. Students simplify complicated expressions into simple terms. Students recognize the effects of transformations and describe them in terms of congruence and similarity.

## RATIOS AND PROPORTIONAL RELATIONSHIPS

### Analyze proportional relationships and use them to solve real-world and mathematical problems.

**7.RP.A.1 Compute unit rates, including those involving complex fractions, with like or different units.**

In addition to Proficient, the **Advanced** student is able to compute **multi-step** unit rates, including those involving complex fractions, with like or different units.

The **Proficient** student is able to compute unit rates, including those involving complex fractions, with like or different units.

The **Basic** student is able to compute unit rates, including those involving complex fractions, with like units.

The **Below Basic** student may be able to compute unit rates, including those involving integers, with like units.

**7.RP.A.2 Recognize and represent proportional relationships between quantities.**

**7.RP.A.2A** Decide whether two quantities in a table or graph are in a proportional relationship.

**7.RP.A.2B** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

**7.RP.A.2C** Represent proportional relationships with equations.

**7.RP.A.2D** Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where $r$ is the unit rate.

In addition to Proficient, the **Advanced** student is able to recognize and represent proportional relationships between quantities.

A. Justify proportionality in a table or graph by extrapolating and/or interpolating ratios that fit the proportional relationship.

B. Compare the constant of proportionality (unit rate) when given two different representations (tables, graphs, equations, diagrams, and verbal descriptions).

C. Create a scenario with a real-world context that represents a given proportional equation.

D. Extrapolate or interpolate coordinates of another point which follows the proportional relationship and explain the reasoning.

The **Proficient** student is able to recognize and represent proportional relationships between quantities.

A. Decide whether two quantities in a table or graph are in a proportional relationship.

B. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

C. Represent proportional relationships with equations.

D. Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where $r$ is the unit rate.

The **Basic** student is able to recognize and represent proportional relationships between quantities.

A. Identify characteristics of proportionality from a proportional table (consistent unit rates and passes through the origin).
B. Calculate the constant of proportionality (unit rate) from a verbal descriptions of a proportional relationship.
C. Represent proportional relationships with equations when the unit rate is given.
D. Identify the unit rate on the graph of a proportional relationship when given the coordinate $(1, r)$ where $r$ is the unit rate.

The **Below Basic** student may be able to recognize and represent proportional relationships between quantities.

A. Identify characteristics of proportionality from a proportional graph (linear and passing through the origin).
B. Identify that the unit rate and the constant of proportionality are the same value.
C. Identify an equation that represents a proportional relationship.
D. Identify coordinates on a graph in the proportional relationship.

**7.RP.A.3 Solve multi-step real-world and mathematical problems involving ratios and percentages.**

In addition to Proficient, the **Advanced** student is able to represent and solve multi-step real-world and mathematical problems involving ratios and percentages in multiple ways.

The **Proficient** student is able to solve multi-step real-world and mathematical problems involving ratios and percentages (e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error).

The **Basic** student is able to solve one-step real-world and mathematical problems involving ratios and percentages.

The **Below Basic** student may be able to identify a proportion that can be used to solve real-world or mathematical problems involving ratios and percentages.

## THE NUMBER SYSTEM

### Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

**7.NS.B.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers.**

**7.NS.B.1A** Describe situations in which opposite quantities combine to make 0 (the additive identity).

**7.NS.B.1B** Understand that $p + q$ represents the distance $|q|$ from $p$ whose placement is determined by the sign of $q$. Interpret sums of rational numbers by describing real-world contexts.

**7.NS.B.1C** Show that a number and its opposite have a sum of 0 (are additive inverses).

**7.NS.B.1D** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Apply this principal in real-world contexts.

**7.NS.B.1E** Apply properties of addition as strategies to add and subtract rational numbers.

In addition to Proficient, the **Advanced** student is able to add and subtract rational numbers showing more than one strategy (e.g., number line diagrams, subtraction as adding the additive inverse, properties of addition).

The **Proficient** student is able to add and subtract two rational numbers and interpret sums of rational numbers by describing them in real-world contexts.

The **Basic** student is able to add and subtract two integers.

The **Below Basic** student may be able to add and subtract two integers given a number line or manipulatives.

**7.NS.B.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.**

**7.NS.B.2A** 1. Understand that the multiplicative inverse of a number is its reciprocal and their product is equal to one (the multiplicative identity). 2. Understand positive and negative sign rules for multiplying rational numbers. Interpret products of rational numbers by describing real-world contexts.

**7.NS.B.2B** Understand that integers can be divided, provided that the divisor is not 0, and every quotient of integers is a rational number. Recognize that if p and q are integers then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

**7.NS.B.2C** Apply properties of multiplication (Commutative, Associative, Distributive, or Properties of Identity and Inverse elements) to multiply and divide rational numbers.

**7.NS.B.2D** Convert a rational number to a decimal. Recognize that rational numbers can be written as fractions or decimal numbers that terminate or repeat.

In addition to Proficient, the **Advanced** student is able to multiply and divide rational numbers showing more than one strategy (e.g., as repeated addition, as repeated subtraction, on a number line, properties of multiplication).

The **Proficient** student is able to multiply and divide two rational numbers and interpret products and quotients of rational numbers by describing real-world contexts.

- Understand that integers can be divided, provided that the divisor is not 0. Recognize that if p and q are integers then $-(p/q) = (-p)/q = p/(-q)$.
- Convert a rational number to a decimal. Recognize that rational numbers can be written as fractions or decimal numbers that terminate or repeat.

The **Basic** student is able to multiply and divide two integers. Convert a rational number to a decimal and recognize terminating or repeating decimals.

The **Below Basic** student may be able to multiply and divide two integers given a number line or manipulatives.

**7.NS.B.3 Solve real-world and mathematical problems involving the four arithmetic operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)**

In addition to Proficient, the **Advanced** student is able to solve multi-step real-world and mathematical problems involving the four arithmetic operations with different representations of rational numbers (fractions, decimals, percentages, or integers).

The **Proficient** student is able to solve real-world and mathematical problems involving the four arithmetic operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

The **Basic** student is able to solve real-world and mathematical problems involving the four arithmetic operations with integers.

The **Below Basic** student may be able to determine if an answer will be positive or negative in a one- or two- step real-world or mathematical problem involving the four arithmetic operations with integers.

## EXPRESSIONS AND EQUATIONS

### Use properties of operations to generate equivalent expressions.

**7.EE.C.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.**

In addition to Proficient, the **Advanced** student is able to create more than one equivalent expression when adding, subtracting, factoring, and expanding rational linear expressions and justify equivalence based on properties or a visual representation.

The **Proficient** student is able to add, subtract, factor, and expand linear expressions with rational coefficients.

The **Basic** student is able to combine like terms or use the Distributive Property to simplify integer expressions.

The **Below Basic** student may be able to identify like terms in an expression.

**7.EE.C.2 Recognize that algebraic expressions may have a variety of equivalent forms that reveal different information, and determine an appropriate form for a given real-world situation.**

In addition to Proficient, the **Advanced** student is able to write different equivalent forms from real-world situations and explain why they are equivalent, referring to what the value represents for each term based on the context of the problem.
The **Proficient** student is able to identify equivalent forms of expressions from real-world situations, and interpret what the value represents for each term based on the context of the problem.
The **Basic** student is able to interpret what the value represents for each term based on the context of a real-world problem.
The **Below Basic** student may be able to tell what the variable represents in the context of a real-world problem in an algebraic expression.

## Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

**7.EE.D.3 Solve multi-step real-world and mathematical problems involving rational numbers. Include fraction bars as a grouping symbol.**

In addition to Proficient, the **Advanced** student is able to recognize a numerical expression that could be used to solve a multi-step real-world or mathematical problem involving rational numbers. Include fraction bars as a grouping symbol.
The **Proficient** student is able to solve multi-step real-world and mathematical problems involving rational numbers. Include fraction bars as a grouping symbol.
The **Basic** student is able to solve two-step real-world and mathematical problems involving rational numbers. Include fraction bars as a grouping symbol.
The **Below Basic** student may be able to solve two-step real-world and mathematical problems involving integer values.

**7.EE.D.4 Apply the concepts of linear equations and inequalities in one variable to real-world and mathematical situations.**

**7.EE.D.4A** Write and fluently solve linear equations of the form $ax + b = c$ *and* $a(x + b) = c$ where $a$, $b$, and $c$ are rational numbers.
**7.EE.D.4B** Write and solve multi-step linear equations that include the use of the Distributive Property and combining like terms. Exclude equations that contain variables on both sides.
**7.EE.D.4C** Write and solve two-step linear inequalities. Graph the solution set on a number line and interpret its meaning.
**7.EE.D.4D** Identify and justify the steps for solving multi-step linear equations and two-step linear inequalities.

In addition to Proficient, the **Advanced** student is able to apply the concepts of linear equations and inequalities in one variable to real-world and mathematical situations.

A. Write and fluently solve linear equations of the form $ax + b = c$ and $a(x + b) = c$ where $a$, $b$, and $c$ are rational numbers. Recognize and verify other equivalent forms of the equation.
B. Create a real-world scenario when given a multi-step linear equation that includes the use of the Distributive Property and combining like terms.
C. Write and solve a multi-step linear inequality that includes the use of the Distributive Property or combining like terms. Graph the solution set on a number line and interpret its meaning.
D. Identify and justify multiple ways for solving multi-step linear equations and inequalities that include the use of the Distributive Property and combining like terms.

The **Proficient** student is able to apply the concepts of linear equations and inequalities in one variable to real-world and mathematical situations.

A. Write and fluently solve linear equations of the form $ax + b = c$ and $a(x + b) = c$ where $a$, $b$, and $c$ are rational numbers.
B. Write and solve multi-step linear equations that include the use of the Distributive Property and combining like terms. Exclude equations that contain variables on both sides.
C. Write and solve two-step linear inequalities. Graph the solution set on a number line and interpret its meaning.

D. Identify and justify the steps for solving multi-step linear equations and two-step linear inequalities.

The **Basic** student is able to apply the concepts of linear equations and inequalities in one variable to real-world and mathematical situations.

A. Write and fluently solve linear equations of the form $ax + b = c$ and $a(x + b) = c$ where $a$, $b$, and $c$ are integers.

B. Solve multi-step linear equations that include the use of the Distributive Property or combining like terms. Exclude equations that contain variables on both sides.

C. Solve two-step linear inequalities. Graph the solution set on a number line.

D. Identify and justify the steps for solving two-step linear equations and two-step linear inequalities with whole numbers.

The **Below Basic** student may be able to apply the concepts of linear equations and inequalities in one variable to real-world and mathematical situations.

A. Write and solve one-step linear equations with whole numbers.

B. Use the Distributive Property or combine like terms when given a multi-step linear equation to simplify the equation. Exclude equations that contain variables on both sides.

C. Solve one-step linear inequalities with whole numbers. Graph the solution set on a number line.

D. Identify and justify the steps for solving one-step linear equations and one-step linear inequalities with whole numbers.

## GEOMETRY

### Draw, construct, and describe geometrical figures and describe the relationships between them.

**7.G.E.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing.**

In addition to Proficient, the **Advanced** student is able to identify and reproduce scale drawing(s) at different scales with respect to the dimensions of the actual figure.

The **Proficient** student is able to solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing.

The **Basic** student is able to compute a single dimension from a scale drawing of a geometric figure.

The **Below Basic** student may be able to compute a single dimension from a scale drawing of a simple geometric figure with a scale factor of 2, 3, 5, or 10 when given a visual representation.

**7.G.E.2 Draw geometric shapes with given conditions using a variety of tools (e.g., ruler and protractor, or technology). Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.**

In addition to Proficient, the **Advanced** student is able to explain the conditions for a unique triangle, more than one triangle, or no triangle.

The **Proficient** student is able to draw geometric shapes with given conditions using a variety of tools (e.g., ruler and protractor, or technology). Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

The **Basic** student is able to draw a triangle using a variety of tools when given one angle measure or one side length.

The **Below Basic** student may be able to identify the type of triangle with respect to angle measures and side measures.

**7.G.E.3 Describe the two-dimensional figures that result from slicing three-dimensional figures parallel to the base, as in plane sections of right rectangular prisms and right rectangular pyramids.**

In addition to Proficient, the **Advanced** student is able to describe the two-dimensional figure that results from slicing a three-dimensional figure parallel, perpendicular, or oblique to the base as in plane sections of right rectangular prisms, right rectangular pyramids, or cylinders.
The **Proficient** student is able to describe the two-dimensional figure (shape and size in relation to the base) that results from slicing a three-dimensional figure parallel to the base, as in plane sections of right rectangular prisms and right rectangular pyramids.
The **Basic** student is able to identify the two-dimensional figure that result from slicing a three-dimensional figure parallel to the base, as in plane sections of right rectangular prisms and right rectangular pyramids when given a visual representation.
The **Below Basic** student may be able to identify the two-dimensional figure that result from slicing a three-dimensional figure parallel to the base, as in plane sections of right rectangular prisms when given a visual representation where the cross-section is drawn.

## Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

**7.G.F.4 Investigate the concept of circles.**

**7.G.F.4A** Demonstrate an understanding of the proportional relationships between diameter, radius, and circumference of a circle.
**7.G.F.4B** Understand that $\pi$ is defined by the constant of proportionality between the circumference and diameter.
**7.G.F.4C** Given the formulas for circumference and area of circles, solve real-world and mathematical problems.

In addition to Proficient, the **Advanced** student is able to determine how much area or circumference changes based on a change in radius or diameter.
**Assessment Boundary:** Specify if calculations should be performed with 3.14 or the $\pi$ button. Specify the place value to which answers must be rounded. Do not include solutions in terms of $\pi$.
The **Proficient** student is able to investigate the concept of circles.

A. Demonstrate an understanding of the proportional relationships between diameter, radius, and circumference of a circle.
B. Understand that $\pi$ is defined by the constant of proportionality between the circumference and diameter.
C. Given the formulas for circumference and area of circles, solve real-world and mathematical problems.

**Assessment Boundary:** Specify if calculations should be performed with 3.14 or the $\pi$ button. Specify the place value to which answers must be rounded. Do not include solutions in terms of $\pi$.
The **Basic** student is able to investigate the concept of circles.

A. Find radius when given the diameter and find diameter when given radius.
C. Given the formulas for circumference and area of circles, solve mathematical problems.

**Assessment Boundary:** Specify if calculations should be performed with 3.14 or the $\pi$ button. Specify the place value to which answers must be rounded. Do not include solutions in terms of $\pi$.
The **Below Basic** student may be able to investigate the concept of circles.

A. Identify parts of a circle (radius, diameter, center, and circumference).
C. Given a word problem, identify which formula would be used to solve the problem (area or circumference).

**Assessment Boundary:** Specify if calculations should be performed with 3.14 or the $\pi$ button. Specify the place value to which answers must be rounded. Do not include solutions in terms of $\pi$.

**7.G.F.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.**

In addition to Proficient, the **Advanced** student is able to write and solve equations for unknown angles in a complex diagram using facts about supplementary, complementary, vertical, and adjacent angles.
The **Proficient** student is able to use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
The **Basic** student is able to use facts about supplementary, complementary, and vertical angles to find missing angle measurements given a verbal description or visual representation.
The **Below Basic** student may be able to identify supplementary, complementary, vertical, and adjacent angles when given a visual representation.

**7.G.F.6 Solve real-world and mathematical problems involving.**
**7.G.F.6A** Area and surface area of objects composed of triangles and quadrilaterals;
**7.G.F.6B** Volume of objects composed only of right prisms having triangular or quadrilateral bases.
In addition to Proficient, the **Advanced** student is able to solve real-world and mathematical problems involving:
A. i. Find a missing dimension when given the area of objects composed of triangles, quadrilaterals, circles, and semi-circles.
ii. Find a missing dimension when given the surface area of objects composed of triangles and quadrilaterals.
B. Find a missing dimension when given the volume of objects composed only of right prisms having triangular or quadrilateral bases.
The **Proficient** student is able to solve real-world and mathematical problems involving:
A. i. Find a missing dimension when given the area of objects composed of triangles, quadrilaterals, circles, and semi-circles.
ii. Find a missing dimension when given the surface area of objects composed of triangles and quadrilaterals
B. Find a missing dimension when given the volume of objects composed only of right prisms having triangular or quadrilateral bases.
The **Basic** student is able to solve real-world and mathematical problems involving:
A. i. Area of objects composed of triangles and/or parallelograms.
ii. Surface area of objects where the base is an equilateral triangle or rectangle.
B. Volume of right prisms having triangular and rectangular bases.
The **Below Basic** student may be able to solve real-world and mathematical problems involving:
A. i. Area of triangles and/or parallelograms.
ii. Surface area of right prisms and/or right pyramids when given the nets with the areas labeled.
B. Volume of right rectangular prisms.

## STATISTICS AND PROBABILITY

### Use random sampling to draw inferences about a population.

**7.SP.G.1 Solve real-world and mathematical problems involving:**

**7.SP.G.1A** Understand that a sample is a subset of a population.
**7.SP.G.1B** Differentiate between random and non-random sampling.
**7.SP.G.1C** Understand that generalizations from a sample are valid only if the sample is representative of the population.
**7.SP.G.1D** Understand that random sampling is used to gather a representative sample and tends to support valid inferences about the population.

In addition to Proficient, the **Advanced** student is able to solve real-world and mathematical problems involving:

A. Generating a sample that is a subset of a population.
B. Creating the parameters for a random sample.
C. Writing a generalization and justifying its validity, given a population and a sample.
D. Analyzing multiple random samples to explain why there might be differences in inferences from the data **or** analyzing errors in data collection performed by others.

The **Proficient** student is able to solve real-world and mathematical problems involving:

A. Describing a sample that is a subset of a population.
B. Differentiating between random and non-random sampling.
C. Determining if a generalization is valid by justifying whether or not the sample is representative of the population.
D. Determining if inferences about the population are valid based on how the given sample was collected.

The **Basic** student is able to solve real-world and mathematical problems involving:

A. Identifying a sample that is a subset of a population.
B. Identifying a random sample.
C. Identifying a generalization based on a sample that is representative of the population.
D. Identifying a random sampling that supports valid inferences about the given population.

The **Below Basic** student may be able to solve real-world and mathematical problems involving:

A. Defining sample or population.
B. Defining random sample.
C. Identifying a representative sample for a given population.
D. Defining valid inference.

**7.SP.G.2 Draw inferences about a population by collecting multiple random samples of the same size to investigate variability in estimates of the characteristic of interest.**

In addition to Proficient, the **Advanced** student is able to draw inferences about a population by collecting multiple random samples of the same size to investigate variability in estimates of the characteristic of interest and justify variability in context of the situation.
The **Proficient** student is able to draw inferences about a population by collecting multiple random samples of the same size to investigate variability in estimates of the characteristic of interest.
The **Basic** student is able to draw inferences about a population given data from a random sample and recognize that random samples produce variability.
The **Below Basic** student may be able to define variability.

## Draw informal comparative inferences about two populations.

**7.SP.H.3 Visually compare the centers, spreads, and overlap of two displays of data (e.g., back-to-back stem and leaf plots, dot plots, histograms, box plots) that are graphed on the same scale and draw inferences about this data.**

In addition to Proficient, the **Advanced** student is able to visually compare the centers, spreads, and overlap of two displays of data (e.g., back-to-back stem and leaf plots, dot plots, histograms, box plots) that are graphed on different scales or in different representations and draw inferences about this data.

The **Proficient** student is able to visually compare the centers, spreads, and overlap of two displays of data (e.g., back-to-back stem and leaf plots, dot plots, histograms, box plots) that are graphed on the same scale and draw inferences about this data.

The **Basic** student is able to visually compare the centers and spreads of two displays of data (e.g., dot plots, box plots) that are graphed on the same scale and draw inferences about this data.

The **Below Basic** student may be able to visually compare the centers and spreads of two box plots that are graphed on the same scale.

**7.SP.H.4 Given measures of center and variability (mean, median and/or mode; range, interquartile range, and/or standard deviation), for numerical data from random samples, draw appropriate informal comparative inferences about two populations.**

In addition to Proficient, the **Advanced** student is able to given measures of center and variability (mean, median, and/or mode; range, interquartile range, and/or standard deviation), for numerical data from random samples, draw appropriate informal comparative inferences about multiple populations and determine to which population(s) given values are most likely to correspond.

The **Proficient** student is able to given measures of center and variability (mean, median, and/or mode; range, interquartile range, and/or standard deviation), for numerical data from random samples, draw appropriate informal comparative inferences about two populations.

The **Basic** student is able to given measures of center and variability (mean and/or median; range), for numerical data from random samples, identify valid comparisons about two populations.

The **Below Basic** student may be able to identify measures of center and variability.

## Investigate chance processes and develop, use, and evaluate probability models.

**7.SP.I.5 Find and interpret the probability of a random event. Understand that the probability of a random event is a number between, and including, 0 and 1 that expresses the likelihood of the event occurring.**

In addition to Proficient, the **Advanced** student is able to justify why probability cannot be greater than 1 or less than 0.

The **Proficient** student is able to find and interpret the probability of a random event. Understand that the probability of a random event is a number between, and including, 0 and 1 that expresses the likelihood of the event occurring.

The **Basic** student is able to interpret the meaning of a given probability.

The **Below Basic** student may be able to categorize events using certain, likely, equal chance, unlikely, or impossible.

**7.SP.I.6 Collect multiple samples to compare the relationship between theoretical and experimental probabilities for simple events.**

In addition to Proficient, the **Advanced** student is able to recognize and justify why the experimental probability approaches the theoretical probability as the relative frequency of an event increases.

The **Proficient** student is able to collect multiple samples to compare the relationship between theoretical and experimental probabilities for simple events.

The **Basic** student is able to collect one sample to compare the relationship between theoretical and experimental probabilities for a simple event.

The **Below Basic** student may be able to identify theoretical and experimental probabilities for a simple event.

**7.SP.I.7 Apply the concepts of theoretical and experimental probabilities for simple events.**

**7.SP.I.7A** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

**7.SP.I.7B** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

**7.SP.I.7C** Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancies.

In addition to Proficient, the **Advanced** student is able to given probabilities or outcomes, create the corresponding probability experiment.

The **Proficient** student is able to apply the concepts of theoretical and experimental probabilities for simple events.

A. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

B. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

C. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancies.

The **Basic** student is able to given the theoretical and experimental probabilities from a model, compare probabilities; if the agreement is not good, explain possible sources of the discrepancies.

The **Below Basic** student may be able to explain the difference between experimental and theoretical probability.

**7.SP.I.8 Find probabilities of compound events using organized lists, tables, and tree diagrams.**

**7.SP.I.8A** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

**7.SP.I.8B** Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

In addition to Proficient, the **Advanced** student is able to find the probabilities of compound dependent events.

The **Proficient** student is able to find probabilities of compound events using organized lists, tables, and tree diagrams.

A. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

B. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

**Assessment Boundary:** Only include independent events.

The **Basic** student is able to complete an organized list, a table, or a tree diagram and find the probability of a compound event.

**Assessment Boundary:** Only include independent events.

The **Below Basic** student may be able to find the probability of a compound event when given the sample space.

**Assessment Boundary:** Only include independent events.

**Companion document to the 2018 Mathematics Content Standards**

# Grade 8 Mathematics Content Standards & PLDs

## GRADE 8 MATH PRACTICES

### MP1 Make sense of problems and persevere in solving them.

**8.MP.1** In grade 8, students solve real-world problems through the application of algebraic and geometric concepts and discuss (verbally or in writing) how they solve them. Students analyze the problem (including what is given, not given, and what is being asked), identify what strategies are needed, choose the most efficient pathway, then make an initial attempt to solve the problem. Students analyze the result for validity and refine strategies if necessary.

### MP2 Reason abstractly and quantitatively.

**8.MP.2** Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

### MP3 Construct viable arguments and critique the reasoning of others.

**8.MP.3** Students construct arguments using verbal or written explanations that involve solving problems with real numbers. They make conjectures, explore validity, reason mathematically, justify, evaluate their own thinking and analytically critique the reasoning of other students.

### MP4 Model with mathematics.

**8.MP.4** Students can clearly show their work by using diagrams, words, symbols or pictures. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results of problems involving real numbers in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

### MP5 Use appropriate tools strategically.

**8.MP.5** Students consider available tools (including estimation, concrete models, and technology as appropriate), and decide when certain tools might be helpful. Students can interpret results provided by technology. They choose the representation (table, graph, equation, words) that best suits the problem. Students use concrete models to develop insight into linear equations and other concepts. Students then extend this insight to more abstract representations, including pictures and symbols. Students understand the limitations of each tool. Tools might include: integer tiles, algebra tiles, geometric nets, number lines, graphing technology, scientific calculator, paper and pencil, and others.

### MP6 Attend to precision.

**8.MP.6** Students continue to refine their mathematical communication skills by using clear and precise mathematical language in their discussions with others and in their own reasoning. Students define variables, including their relationship, specify units of measure, and label each axis accurately. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays. Students use appropriate symbols, labels, and units of measure when solving problems with calculations that are accurate and efficient. Answer to the problem matches what was asked in the problem.

### MP7 Look for and make use of structure.

**8.MP.7** Students routinely seek patterns or structure to model and solve problems. They apply properties to solve problems based upon patterns they have identified. Students examine patterns to generate equations and describe relationships. Students simplify complicated expressions into simple terms. Students recognize the effects of transformations and describe them in terms of congruence and similarity.

### MP8 Look for and express regularity in repeated reasoning.

**8.MP.8** Students use repeated reasoning to understand algorithms and make generalizations about patterns. They develop efficient strategies for solving problems and check for reasonableness of answers. Students ask questions such as, "What evidence supports that conclusion?"

## THE NUMBER SYSTEM

### Know that there are numbers that are not rational, and approximate them by rational numbers.

**8.NS.A.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. Explore the real number system and its appropriate usage in real-world situations.**

**8.NS.A.1A** Make comparisons between rational and irrational numbers.
**8.NS.A.1B** Understand that all real numbers have a decimal expansion.
**8.NS.A.1C** Model the hierarchy of the real number system, including natural, whole, integer, rational, and irrational numbers.
**8.NS.A.1D** Convert repeating decimals to fractions.

In addition to Proficient, the **Advanced** student is able to compare and contrast properties of rational and irrational numbers.

The **Proficient** student is able to know that numbers that are not rational are called irrational. Show that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. Explore the real number system and its appropriate usage in real-world situations.

A. Make comparisons between rational and irrational numbers.
B. Show that real numbers (excluding irrational numbers) have a decimal expansion.
C. Model the hierarchy of the real number system, including natural, whole, integer, rational, and irrational numbers.
D. Convert repeating decimals to fractions.

The **Basic** student is able to know that numbers that are not rational are called irrational. Show that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. Explore the real number system and its appropriate usage in real-world situations.

A. Make comparisons between rational and irrational numbers.
B. Show that real numbers (excluding irrational numbers) have a decimal expansion.
C. Model the hierarchy of the real number system, including natural, whole, integer, rational, and irrational numbers.

The **Below Basic** student may be able to know that numbers that are not rational are called irrational. Show that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. Explore the real number system and its appropriate usage in real-world situations. Make comparisons between rational and irrational numbers.

**8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.**

In addition to Proficient, the **Advanced** student is able to using estimation strategies, order a set of irrational numbers and explain your reasoning.
The **Proficient** student is able to use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.
The **Basic** student is able to use rational approximations to locate irrational numbers on a number line and estimate the value of expressions.
The **Below Basic** student may be able to use rational approximations to locate irrational numbers on a number line.

## EXPRESSIONS AND EQUATIONS

### Work with radicals and integer exponents.

**8.EE.B.1 Understand and apply the Laws of Exponents (i.e. Product Rule, Quotient Rule, Power to a Power, Product to a Power, Quotient to a Power, Zero Power Property, negative exponents) to generate equivalent numerical expressions limited to integer exponents.**

In addition to Proficient, the **Advanced** student is able to apply all of the following Laws of exponents to generate equivalent algebraic expressions limited Eto integer exponents.

- Product Rule.
- Quotient Rule.
- Power to a Power.
- Product to a Power.
- Quotient to a Power
- Zero Power Property.
- Negative Exponents.

**Assessment Boundary:** Limit to no more than two rules per problem.

The **Proficient** student is able to apply all of the following Laws of Exponents to generate equivalent numerical expressions limited to integer exponents.

- Product Rule.
- Quotient Rule.
- Power to a Power.
- Product to a Power.
- Quotient to a Power.
- Zero Power Property.
- Negative Exponents.

The **Basic** student is able to apply all of the following Laws of Exponents to generate equivalent numerical expressions limited to integer exponents.

- Product Rule.
- Quotient Rule.
- Power to a Power.
- Zero Power Property.

The **Below Basic** student may be able to apply all of the following Laws of Exponents to generate equivalent numerical expressions limited to integer exponents.

- Product Rule.
- Quotient Rule.

**8.EE.B.2 Investigate concepts of square and cube roots.**

**Assessment Boundary:** Include perfect squares up to 144 and perfect cubes up to 125.

**8.EE.B.2A** Use radical notation, if applicable, to represent the exact solutions to equations of the form $x^2 = p$ and $x^3 = q$ where p is a positive rational number and q is any rational number.

**8.EE.B.2B** Evaluate square roots of small perfect squares and cube roots of small perfect cubes.

**8.EE.B.2C** Recognize that square roots of non-perfect squares and the cube roots of non-perfect cubes are irrational.

In addition to Proficient, the **Advanced** student is able to investigate concepts of square and cube roots. Use radical notation, if applicable, to represent the exact solutions to equations of the form $x^2 + a = p$ and $x^3 + a = q$ where a, p, and q are positive rational numbers such that $p - a$ and $q - a$ are greater than or equal to zero.

**Assessment Boundary:** Include perfect squares up to 144 and perfect cubes up to 125.

The **Proficient** student is able to investigate concepts of square and cube roots.

A. Use radical notation, if applicable, to represent the exact solutions to equations of the form $x^2 = p$ and $x^3 = q$ where $p$ is a positive rational number and $q$ is any rational number.

B. Evaluate square roots of small perfect squares and cube roots of small perfect cubes.

C. Recognize that square roots of non-perfect squares and the cube roots of non-perfect cubes are irrational.

**Assessment Boundary:** Include perfect squares up to 144 and perfect cubes up to 125.

The **Basic** student is able to investigate concepts of square and cube roots.

A. Use radical notation, if applicable, to represent the exact solutions to equations of the form $x^2 = p$ and $x^3 = q$ where p is a positive rational number and q is any rational number.

B. Evaluate square roots of small perfect squares and cube roots of small perfect cubes.

**Assessment Boundary:** Include perfect squares up to 144 and perfect cubes up to 125.

The **Below Basic** student may be able to evaluate square roots of small perfect squares and cube roots of small perfect cubes.

**Assessment Boundary:** Include perfect squares up to 144 and perfect cubes up to 125.

**8.EE.B.3 Explore the relationship between quantities in decimal and scientific notation.**

**8.EE.B.3A** Express very large and very small quantities, $p$, in scientific notation in the form $a \cdot 10^b = p$ where $1 \leq a < 10$ and b is an integer.

**8.EE.B.3B** Translate between decimal notation and scientific notation.

**8.EE.B.3C** Estimate and compare the relative size of two quantities in scientific notation.

In addition to Proficient, the **Advanced** student is able to compare the relative size of two quantities written in decimal and scientific notation in a real-world context.

The **Proficient** student is able to explore the relationship between quantities in decimal and scientific notation.

A. Express very large and very small quantities, p, in scientific notation in the form $a \cdot 10^b = p$ where $1 \leq a < 10$ and b is an integer.

B. Translate between decimal notation and scientific notation.

C. Estimate and compare the relative size of two quantities in scientific notation.

The **Basic** student is able to explore the relationship between quantities in decimal and scientific notation.

A. Express very large and very small quantities, p, in scientific notation in the form $a \cdot 10^b = p$ where $1 \leq a < 10$ and b is an integer.
B. Translate between decimal notation and scientific notation.

The **Below Basic** student may be able to express very large and very small quantities, p, in scientific notation in the form $a \cdot 10^b = p$ where $1 \leq a < 10$ and b is an integer.

**8.EE.B.4 Apply the concepts of decimal and scientific notation to real-world and mathematical problems.**

**8.EE.B.4A** Select appropriate units of measure when representing answers in scientific notation.

**8.EE.B.4B** Interpret scientific notation that has been generated by a variety of technologies.

In addition to Proficient, the **Advanced** student is able to select appropriate units of measure when multiplying and dividing numbers written in scientific notation in real-world and mathematical problems.

The **Proficient** student is able to apply the concepts of decimal and scientific notation to real-world and mathematical problems.

A. Select appropriate units of measure when representing answers in scientific notation.
B. Interpret scientific notation that has been generated by a variety of technologies.

The **Basic** student is able to select appropriate units of measure when representing answers in decimal and scientific notation in real-world and mathematical problems.

The **Below Basic** student may be able to select appropriate units of measure when representing answers in scientific notation in real-world and mathematical problems.

## Understand the connections between proportional relationships, lines, and linear equations.

**8.EE.C.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.**

In addition to Proficient, the **Advanced** student is able to create a table, graph, and equation of a proportional relationship given a written description.

The **Proficient** student is able to graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

The **Basic** student is able to graph proportional relationships, interpreting the unit rate as the slope of the graph.

The **Below Basic** student may be able to graph proportional relationships from a table of values.

**8.EE.C.6 Explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y =mx for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $(0, b)$.**

In addition to Proficient, the **Advanced** student is able to derive the equation $y = mx + b$ for a line intercepting the vertical axis at $(0, b)$ from a written description.

The **Proficient** student is able to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $(0, b)$.

The **Basic** student is able to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin.

The **Below Basic** student may be able to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane.

## Analyze and solve linear equations and pairs of simultaneous linear equations.

**8.EE.D.7 Extend concepts of linear equations and inequalities in one variable to more complex multi-step equations and inequalities in real-world and mathematical situations.**

**8.EE.D.7A** Solve linear equations and inequalities with rational number coefficients that include the use of the Distributive Property, combining like terms, and variable terms on both sides.

**8.EE.D.7B** Recognize the three types of solutions to linear equations: one solution, infinitely many solutions, or no solutions.

**8.EE.D.7C** Generate linear equations with the three types of solutions.

**8.EE.D.7D** Justify why linear equations have a specific type of solution.

In addition to Proficient, the **Advanced** student is able to create and solve a multi-step equation or inequality in a real-world context given a written description.

The **Proficient** student is able to extend concepts of linear equations and inequalities in one variable to more complex multi-step equations and inequalities in real-world and mathematical situations.

A. Solve linear equations and inequalities with rational number coefficients that include the use of the Distributive Property, combining like terms, and variable terms on both sides.
B. Recognize the three types of solutions to linear equations: one solution, infinitely many solutions, or no solutions.
C. Generate linear equations with the three types of solutions.
D. Justify why linear equations have a specific type of solution.

The **Basic** student is able to extend concepts of linear equations and inequalities in one variable to more complex multi-step equations and inequalities in real-world and mathematical situations.

A. Solve linear equations and inequalities with rational number coefficients that include the use of the Distributive Property, combining like terms, and variable terms on both sides.
B. Recognize the three types of solutions to linear equations: one solution, infinitely many solutions, or no solutions.

The **Below Basic** student may be able to solve linear equations and inequalities with rational number coefficients that include the use of the Distributive Property, combining like terms, and variable terms on one side.

**8.EE.D.8 Analyze and solve pairs of simultaneous linear equations.**

**8.EE.D.8A** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

**8.EE.D.8B** Solve systems of two linear equations in two variables with integer solutions by graphing the equations.

**8.EE.D.8C** Solve simple real-world and mathematical problems leading to two linear equations in two variables given $y = mx + b$ form with integer solutions.

In addition to Proficient, the **Advanced** student is able to create and solve systems of two linear equations from a real-world context that requires simplification to write in the form $y = mx + b$.

The **Proficient** student is able to analyze and solve a system of linear equations.

A. Show that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously, including systems with one, infinitely many, and no solutions.
B. Solve systems of two linear equations in two variables with integer solutions by graphing the equations.
C. Solve simple real-world and mathematical problems leading to two linear equations in two variables given $y = mx + b$ form with integer solutions.

The **Basic** student is able to analyze and solve a system of linear equations.

A. Show that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously, including systems with one, infinitely many, and no solutions.
B. Solve systems of two linear equations in two variables with integer solutions by graphing the equations.

The **Below Basic** student may be able to given a graph, identify the solution to a system of two linear equations, including systems with one, infinitely many, and no solutions.

## FUNCTIONS

### Define, evaluate, and compare functions.

**8.F.E.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.**

In addition to Proficient, the **Advanced** student is able to compare relations in different forms to determine if they represent a function. (Function notation is not required in Grade 8.)
The **Proficient** student is able to determine if a relation represented by a graph, a table, a mapping diagram, and a set of ordered pairs is a function. (Function notation is not required in Grade 8.)
The **Basic** student is able to determine that a relation represented by a table or a set of ordered pairs is a function by demonstrating each input has exactly one output. (Function notation is not required in Grade 8.)
The **Below Basic** student may be able to recognize the input and output values of a relation in a table or a set of ordered pairs. (Function notation is not required in Grade 8.)

**8.F.E.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).**

In addition to Proficient, the **Advanced** student is able to compare properties (intercepts, domain, and range) of one linear function and one non-linear function each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
The **Proficient** student is able to compare properties (intercepts, domain, and range) of two linear functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
The **Basic** student is able to compare properties (intercepts, domain, and range) of two linear functions each represented graphically or numerically in tables.
The **Below Basic** student may be able to compare the domains of two linear functions each represented graphically or numerically in tables.

**8.F.E.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.**

In addition to Proficient, the **Advanced** student is able to identify linear and non-linear functions represented by verbal or written descriptions.
The **Proficient** student is able to recognize the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; write examples of equations and sketch graphs of functions that are not linear.
The **Basic** student is able to recognize the equation $y = mx + b$ as defining a linear function, whose graph is a straight line and sketch graphs of functions that are not linear.
The **Below Basic** student may be able to recognize the equation $y = mx + b$ as defining a linear function, whose graph is a straight line.

## Use functions to model relationships between quantities.

**8.F.F.4 Apply the concepts of linear functions to real-world and mathematical situations.**

**8.F.F.4A** Understand that the slope is the constant rate of change and the y-intercept is the point where $x = 0$.

**8.F.F.4B** Determine the slope and the y-intercept of a linear function given multiple representations, including two points, tables, graphs, equations, and verbal descriptions.

**8.F.F.4C** Construct a function in slope-intercept form that models a linear relationship between two quantities.

**8.F.F.4D** Interpret the meaning of the slope and the y-intercept of a linear function in the context of the situation.

In addition to Proficient, the **Advanced** student is able to apply the concepts of linear functions to real-world and mathematical situations. Analyze the meaning of the slopes and the y-intercepts of two linear functions given as a written description and justify conclusions in the context of the situation.

The **Proficient** student is able to apply the concepts of linear functions to real-world and mathematical situations.

A. Recognize that the slope is the constant rate of change and the y-intercept is the point where $x = 0$ from an equation, graph, table, and verbal description.

B. Determine the slope and the y-intercept of a linear function given multiple representations, including two points, tables, graphs, equations, and verbal descriptions.

C. Construct a function in slope-intercept form that models a linear relationship between two quantities.

D. Interpret the meaning of the slope and the y-intercept of a linear function in the context of the situation.

The **Basic** student is able to apply the concepts of linear functions to real-world and mathematical situations.

A. Recognize that the slope is the constant rate of change and the y-intercept is the point where $x = 0$ from an equation and a graph.

B. Determine the slope and the y-intercept of a linear function given multiple representations, including graphs and equations in slope-intercept form.

C. Identify a function in slope-intercept form that models a linear relationship between two quantities.

D. Interpret the meaning of the slope of a linear function in the context of the situation.

The **Below Basic** student may be able to apply the concepts of linear functions to real-world and mathematical situations.

A. Recognize that the slope is the constant rate of change and the y-intercept is the point where $x = 0$ from a graph.

B. Determine the slope and the y-intercept of a linear function given an equation in slope-intercept form.

C. Match a function in slope-intercept form to the model of a linear relationship between two quantities.

**8.F.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph where the function is increasing, decreasing, constant, linear, or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.**

In addition to Proficient, the **Advanced** student is able to describe qualitatively the functional relationship between two quantities by analyzing a written real-world scenario where the function is increasing, decreasing, constant, linear, or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described in a real-world scenario.

The **Proficient** student is able to describe qualitatively the functional relationship between two quantities by analyzing a graph where the function is increasing, decreasing, constant, linear, or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

The **Basic** student is able to describe qualitatively the functional relationship between two quantities by analyzing a graph where the function is increasing, decreasing, constant, linear, or nonlinear.

The **Below Basic** student may be able to describe qualitatively the functional relationship between two quantities by labeling a graph where the function is increasing, decreasing, constant, linear, or nonlinear when given a word bank.

## GEOMETRY

### Understand congruence and similarity using physical models, transparencies, or geometry software.

**8.G.G.1 Verify experimentally the properties of rotations, reflections, and translations.**

**8.G.G.1A** Lines are taken to lines, and line segments to line segments of the same length.

**8.G.G.1B** Angles are taken to angles of the same measure.

**8.G.G.1C** Parallel lines are taken to parallel lines.

In addition to Proficient, the **Advanced** student is able to verify experimentally the properties of rotations, reflections, and translations.

A. Write a sequence of transformations that takes a line to a line, and line segment to a line segment of the same length.
B. Write a sequence of transformations that takes an angle to an angle of the same measure.
C. Write a sequence of transformations that takes parallel lines to parallel lines.

The **Proficient** student is able to verify experimentally the properties of rotations, reflections, and translations.

A. Demonstrate that lines are taken to lines, and line segments to line segments of the same length.
B. Demonstrate that angles are taken to angles of the same measure.
C. Demonstrate that parallel lines are taken to parallel lines.

The **Basic** student is able to verify experimentally the properties of rotations, reflections, and translations.

A. Select the transformation that shows lines are taken to lines, and line segments to line segments of the same length.
B. Select the transformation that shows angles are taken to angles of the same measure.
C. Select the transformation that shows parallel lines are taken to parallel lines.

The **Below Basic** student may be able to verify experimentally the properties of rotations, reflections, and translations.

A. Select the translation that shows lines are taken to lines, and line segments to line segments of the same length.
B. Select the translation that shows angles are taken to angles of the same measure.
C. Select the translation that shows parallel lines are taken to parallel lines.

**8.G.G.2 Recognize through visual comparison that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.**

In addition to Proficient, the **Advanced** student is able to recognize through visual comparison that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of three or more transformations (rotations, reflections, and translations); given two congruent figures, describe a sequence of three or more transformations that exhibits the congruence between them.

The **Proficient** student is able to recognize through visual comparison that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of at most two transformations (rotations, reflections, and translations); given two congruent figures, describe a sequence of at most two transformations that exhibits the congruence between them.

The **Basic** student is able to recognize through visual comparison that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of at most two transformations (reflections and translations); given two congruent figures, describe a sequence of at most two transformations that exhibits the congruence between them.

The **Below Basic** student may be able to recognize through visual comparison that a two-dimensional figure is congruent to another if the second can be obtained from the first by a transformation (reflection or translation); given two congruent figures, describe a transformation that exhibits the congruence between them.

**8.G.G.3 Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.**

In addition to Proficient, the **Advanced** student is able to describe and justify the sequence of dilations, translations, rotations, and reflections performed on the pre-image to determine the image on a coordinate plane.
The **Proficient** student is able to describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
The **Basic** student is able to describe the effect of dilations, translations, and reflections on two-dimensional figures using coordinates.
The **Below Basic** student may be able to describe the effect of translations and reflections on two-dimensional figures using coordinates.

**8.G.G.4 Recognize through visual comparison that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.**

In addition to Proficient, the **Advanced** student is able to describe and justify a sequence of rotations, reflections, translations, and dilations that maintains similarity between the pre-image and determined image.
The **Proficient** student is able to recognize through visual comparison that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
The **Basic** student is able to recognize through visual comparison that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
The **Below Basic** student may be able to recognize through visual comparison that a two-dimensional figure is similar to another if the second can be obtained from a dilation.

**8.G.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.**

In addition to Proficient, the **Advanced** student is able to use informal arguments to establish that two triangles are similar using the angle-angle criterion for similarity of triangles.
The **Proficient** student is able to use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.
The **Basic** student is able to use informal arguments to establish facts about the angles created when parallel lines are cut by a transversal and the angle-angle criterion for similarity of triangles.
The **Below Basic** student may be able to use informal arguments to establish facts about the angles created when parallel lines are cut by a transversal.

## Understand and apply the Pythagorean Theorem.

**8.G.H.6 Use models or diagrams to explain the Pythagorean Theorem and its converse.**

In addition to Proficient, the **Advanced** student is able to model a proof of the Pythagorean Theorem and its converse using a pictorial representation.
The **Proficient** student is able to use models or diagrams to explain the Pythagorean Theorem and its converse.
The **Basic** student is able to identify the Pythagorean Theorem and its converse.
The **Below Basic** student may be able to identify the Pythagorean Theorem.

**8.G.H.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems.**

In addition to Proficient, the **Advanced** student is able to apply the Pythagorean Theorem to determine unknown side lengths in right triangles in **multi-step** real-world and mathematical problems.

The **Proficient** student is able to apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems.

The **Basic** student is able to apply the Pythagorean Theorem in mathematical problems by setting up the equation $a^2 + b^2 = c^2$ and solving for either leg.

The **Below Basic** student may be able to apply the Pythagorean Theorem in mathematical problems by setting up the equation $a^2 + b^2 = c^2$ and only solving for the hypotenuse.

**8.G.H.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.**

In addition to Proficient, the **Advanced** student is able to apply the distance formula to calculate the distance between two points in a coordinate system.

The **Proficient** student is able to apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

The **Basic** student is able to apply the Pythagorean Theorem to find the distance between two points plotted on a coordinate plane.

The **Below Basic** student may be able to apply the Pythagorean Theorem to find the distance between two points given the right triangle drawn on a coordinate plane.

## Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

**8.G.I.9 Given the formulas, solve real-world and mathematical problems involving volume and surface area of cylinders.**

In addition to Proficient, the **Advanced** student is able to, given the formulas:

- Solve for a component part (radius or height) given the volume of a cylinder. **OR**
- Determine the volume of a cone or sphere. **AND**
- Determine the volume of a composite figure containing two or more cones, cylinders, or spheres.

**Assessment Boundary:** Specify calculations should be performed with the pi button. Limit the place value to up to the thousandths place or written in terms of pi.

The **Proficient** student is able to, given the formulas, solve real-world and mathematical problems involving volume and surface area of cylinders.

**Assessment Boundary:** Specify calculations should be performed with the pi button. Limit the place value to up to the thousandths place or written in terms of pi.

The **Basic** student is able to, given the formulas, solve mathematical problems involving volume and surface area of cylinders.

**Assessment Boundary:** Specify calculations should be performed with the pi button. Limit the place value to up to the thousandths place.

The **Below Basic** student may be able to, given the formulas, solve mathematical problems involving volume of cylinders.

**Assessment Boundary:** Specify calculations should be performed with the pi button. Limit the place value to up to the thousandths place.

## STATISTICS AND PROBABILITY

### Investigate patterns of association in bivariate data.

**8.SP.J.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe the association by form (linear / nonlinear), direction (positive / negative), strength (correlation), and unusual features.**

In addition to Proficient, the **Advanced** student is able to interpret and compare scatter plots for bivariate measurement data by comparing their association by form (linear / nonlinear), direction (positive / negative), strength (correlation), and unusual features.

The **Proficient** student is able to construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe the association by form (linear / nonlinear), direction (positive / negative), strength (correlation), and unusual features.

The **Basic** student is able to construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe the **linear** association by direction (positive / negative) and strength (correlation).

The **Below Basic** student may be able to interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe the **linear** association by direction (positive / negative) and strength (correlation).

**8.SP.J.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.**

In addition to Proficient, the **Advanced** student is able to know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model by plotting the residuals.

The **Proficient** student is able to know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

The **Basic** student is able to know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line.

The **Below Basic** student may be able to know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, identify a line of best fit.

**8.SP.J.3 Use an equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.**

In addition to Proficient, the **Advanced** student is able to draw a line of best fit and create its equation in the context of the bivariate measurement data when given a scatter plot.

The **Proficient** student is able to interpret the slope and y-intercept in the context of the bivariate measurement data when given a scatter plot with a line of best fit and an equation.

The **Basic** student is able to interpret the slope in the context of the bivariate measurement data when given a scatter plot with a line of best fit and an equation.

The **Below Basic** student may be able to match an equation of a line of best fit to bivariate measurement data when given scatter plot with a line of best fit.

**8.SP.J.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.**

**8.SP.J.4A** Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.

**8.SP.J.4B** Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

In addition to Proficient, the **Advanced** student is able to understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table and justifying conclusions about the frequencies and relative frequencies of data in a two-way table.

The **Proficient** student is able to understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

A. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.

B. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

The **Basic** student is able to understand that patterns of association can also be seen in bivariate categorical data by constructing and interpreting a two-way table summarizing data on two categorical variables collected from the same subjects.

The **Below Basic** student may be able to understand that patterns of association can also be seen in bivariate categorical data by completing a two-way table summarizing data on two categorical variables collected from the same subjects.

**Companion document to the 2018 Mathematics Content Standards**

# High School Mathematics Content Standards & PLDs

## GRADE HS MATH PRACTICES

### MP1 Make sense of problems and persevere in solving them.

**HS.MP.1** Students start to examine problems by explaining to themselves the meaning of a problem and restating the problem in their own words. These students analyze the given information in the problem, including constraints, relationships, and goals. Students make conjectures about the form and meaning of the solution, devise a plan, and solve. They will consider both similar problems, and simpler forms of the original problem, in order to gain insight and efficiency in problem solving. Students monitor and evaluate their progress and change course if necessary. Students may utilize algebraic methods or technology. Students explain relationships between equations and the following: descriptions/situations, tables, and graphs. Students produce diagrams of important features and relationships, graph data, and search for patterns or trends. They check answers to problems and continually ask if the solution makes sense in context. They understand different approaches to solving complex problems and identify correspondences between different approaches.

### MP2 Reason abstractly and quantitatively.

**HS.MP.2** Students seek to make sense of quantities and explore relationships in problem situations. Students represent a given situation by defining and manipulating variables. Students consider the units involved and attend to the meaning of quantities in addition to computational reasoning -- knowing and using the different properties of operations.

### MP3 Construct viable arguments and critique the reasoning of others.

**HS.MP.3** Students understand and use stated assumptions, definitions, and previously established results in constructing arguments. Students make conjectures and build logical progressions of statements to explore the truth of their conjectures. They are able to analyze situations through decomposition and produce counterexample(s) if necessary. Students justify their conclusions, communicate these conclusions, and respond to arguments of others. Students make plausible arguments by reasoning inductively about the data and take into account the context from which the data arose. Students are able to compare the effectiveness of two plausible arguments, and distinguish correct logic from flawed logic. If there is a flaw in an argument, then they explain why the logic is flawed. Students determine a general process and/or domain to which an argument applies. The students listen or read the arguments of others, decide whether the argument makes sense, and ask useful questions to clarify or improve the arguments.

### MP4 Model with mathematics.

**HS.MP.4** Students apply their mathematical knowledge to solve problems arising in everyday life, society, and the workplace. Students may use geometry to solve a design problem or they may use a function to describe how one quantity of interest depends on another. Students may use assumptions and approximations to simplify a complicated situation and realize these may need revision later. Students identify important relationships between quantities in a practical situation and map these relationships using tools such as: diagrams, two-way tables, graphs, flowcharts, and formulas. Students analyze those relationships mathematically to draw conclusions and interpret the results in the context of the situation. Students are reflective of the results and may improve the model if it has not served the purpose.

### MP5 Use appropriate tools strategically.

**HS.MP.5** Students consider appropriate tools when solving a mathematical problem, including but not limited to: a) pencil and paper, b) concrete models, c) ruler, d) protractor, e) calculator, f) spreadsheet, and g) analytical software applications. Students familiar with mathematical tools make sound decisions about when each of these tools may be helpful and recognize both the insight to be gained and the limitations of the tool. Students may use a graphing calculator to analyze graphs of functions knowing that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Students may identify relevant external mathematical resources, such as digital content located on a website, and use those resources to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

### MP6 Attend to precision.

**HS.MP.6** Students communicate using mathematically correct definitions in their own reasoning and in discussions with others. They state the meaning of symbols they choose, specify units of measure, and label axes in order to clarify the correspondence with quantities in a problem. Students accurately and efficiently calculate. They express numerical answers with the degree of precision appropriate for the problem context.

### MP7 Look for and make use of structure.

**HS.MP.7** Students look closely to discern a pattern or structure and holistically consider the overview. Students may shift perspectives if needed to gain understanding of the pattern or structure. Students in algebra may use patterns to create equivalent expressions, factor and solve equations, compose functions, and transform figures. They may consider certain algebraic expressions as single objects or as being composed of several objects. Students in geometry recognize the significance of an existing line in a geometric figure and may use the strategy of drawing an auxiliary line for solving problems.

### MP8 Look for and express regularity in repeated reasoning.

**HS.MP.8** Students notice repeated calculations, look for general expressions to annotate the calculation, and consider potential shortcuts. Students maintain oversight of a process as they work to solve problems, derive formulas, or make generalizations, while attending to details. They assess the reasonableness of their intermediate results.

## THE REAL NUMBER SYSTEM

### Extend the properties of exponents to rational exponents.

**N.RN.A.1 Explain how the meaning of the definition of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.**

In addition to Proficient, the **Advanced** student is able to prove, use, and explain the properties of rational exponents (which are an extension of the properties of integer exponents) and extend to real-world context.

The **Proficient** student is able to explain and use the meaning of rational exponents in terms of properties of integer exponents and use proper notation for radicals in terms of rational exponents.

The **Basic** student is able to use proper notation for radicals in terms of rational exponents, but is unable to explain the meaning.

The **Below Basic** student may be able to use proper notation and use structure for integer exponents only.

**N.RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.**

In addition to Proficient, the **Advanced** student is able to compare contexts where radical form is preferable to rational exponents, and vice versa.
The **Proficient** student is able to rewrite expressions involving radicals and rational exponents, using the properties of exponents.
The **Basic** student is able to identify equivalent forms of expressions involving rational exponents (but is not able to rewrite or find the product of multiple radical expressions).
The **Below Basic** student may be able to convert radical notation to rational exponent notation.

### Use properties of rational and irrational numbers.

**N.RN.B.3 Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.**

In addition to Proficient, the **Advanced** student is able to generalize the rules for sum and product properties of rational and irrational numbers.
The **Proficient** student is able to explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
The **Basic** student is able to:

- Explain why the sum or product of rational numbers is rational. **OR**
- That the sum of a rational number and an irrational number is irrational. **OR**
- That the product of a nonzero rational number and an irrational number is irrational.

The **Below Basic** student may be able to explain why adding and multiplying two rational numbers results in a rational number.

## QUANTITIES

### Reason quantitatively and use units to solve problems.

**N.Q.C.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; and choose and interpret the scale and the origin in graphs and data displays.**

In addition to Proficient, the **Advanced** student is able to explain or defend their use of units as a way to understand problems and to guide the solution of multi-step problems; to explain and/or defend their choice of units consistently in formulas; **and/or** to explain and/or defend their choice of the scale and the origin in graphs and data displays.
The **Proficient** student is able to use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; **and** choose and interpret the scale and the origin in graphs and data displays.
The **Basic** student inconsistently uses units as a way to understand problems and/or to guide the solution of problems; chooses and/or interprets units inconsistently in formulas; **and/or** inconsistently chooses and/or interprets the scale and the origin in graphs and data displays.
The **Below Basic** student may need guidance to use units as a way to understand problems and to guide the solution of problems; to choose and interpret units in formulas; **and/or** to choose and interpret the scale and the origin in graphs and data displays.

**N.Q.C.2 Define appropriate quantities for the purpose of descriptive modeling.**

In addition to Proficient, the **Advanced** student is able to explain or defend their choice of quantities for the purpose of descriptive modeling.
The **Proficient** student is able to define appropriate quantities for the purpose of descriptive modeling.
The **Basic** student inconsistently defines quantities for the purpose of descriptive modeling.
The **Below Basic** student may need guidance to define quantities for the purpose of descriptive modeling.

**N.Q.C.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.**

In addition to Proficient, the **Advanced** student is able to explain or defend their choice of level of accuracy appropriate to limitations on measurement when reporting quantities.
The **Proficient** student is able to choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
The **Basic** student inconsistently chooses a level of accuracy appropriate to limitations on measurement when reporting quantities.
The **Below Basic** student needs guidance to choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

## THE COMPLEX NUMBER SYSTEM

### Perform arithmetic operations with complex numbers.

**N.CN.D.1 Know there is a complex number $i$ such that $i^2 = -1$, and every complex number has the form $a + bi$ with $a$ and $b$ real.**

In addition to Proficient, the **Advanced** student is able to make generalizations about the powers of $i$ to write complex numbers in the form $a + bi$ with $a$ and $b$ being real numbers.
The **Proficient** student is able to know there is a complex number $i$ such that $i^2 = -1$, and every complex number has the form $a + bi$ with $a$ and $b$ real.
The **Basic** student incorrectly and/or inconsistently applies $-1$ for $i$ and/or $i^2$.
The **Below Basic** student has a limited understanding of $i^2 = -1$ and needs guidance to correctly use $i$ and/or $i^2$.

**N.CN.D.2 Use the relation $i^2 = -1$ and the Commutative, Associative, and Distributive Properties to add, subtract, and multiply complex numbers.**

In addition to Proficient, the **Advanced** student is able to perform arithmetic operations (add, subtract, multiply, divide) with complex numbers to include other powers of $i$ and $a + bi$.
The **Proficient** student uses the relation $i^2 = -1$ and the Commutative, Associative, and Distributive Properties to add, subtract, and multiply complex numbers.
The **Basic** student inconsistently uses the relation $i^2 = -1$ and the Commutative, Associative, and Distributive Properties to add, subtract, and multiply complex numbers.
The **Below Basic** student inconsistently uses the relation $i^2 = -1$ and the Commutative and Associative Properties to add and subtract complex numbers.

**N.CN.D.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.**

(+) In addition to Proficient, the **Advanced** student is able to simplify complex number expressions that involve a quotient and at least one other operation.
(+) The **Proficient** student is able to find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
(+) The **Basic** student is able to, given the conjugate of a complex number, find the quotients of complex numbers.
(+) The **Below Basic** student does not meet the basic performance level.

### Represent complex numbers and their operations on the complex plane.

**N.CN.E.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.**

(+) In addition to Proficient, the **Advanced** student is able to:

- Given a complex number in rectangular form, convert it to polar form. **AND**
- Given a complex number in polar form, convert it to rectangular form.

(+) The **Proficient** student is able to represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
(+) The **Basic** student is able to represent complex numbers on the complex plane in rectangular form.
(+) The **Below Basic** student does not meet the basic performance level.

**N.CN.E.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120°.**

(+) In addition to Proficient, the **Advanced** student is able to compare and contrast the algebraic and geometric approaches to finding the nth root of all real numbers.
(+) The **Proficient** student is able to represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120°.
(+) The **Basic** student is able to represent addition and subtraction of complex numbers geometrically on the complex plane; use properties of this representation for computation.
(+) The **Below Basic** student does not meet the basic performance level.

**N.CN.E.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.**

(+) In addition to Proficient, the **Advanced** student is able to compare and contrast how distance and midpoint between two numbers are represented on the Cartesian plane versus the complex plane.
(+) The **Proficient** student is able to calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
(+) The **Basic** student is able to calculate the distance between numbers in the complex plane as the modulus of the difference.
(+) The **Below Basic** student does not meet the basic performance level.

## Use complex numbers in polynomial identities and equations.

**N.CN.F.7 Solve quadratic equations with real coefficients that have complex solutions.**

In addition to Proficient, the **Advanced** student is able to:

- Write a quadratic equation with real coefficients in standard form, when given a complex solution (recognizing that complex solutions to quadratic equations come in conjugate pairs). **OR**
- Determine the relationship between the solutions, the discriminant, and the graph of a quadratic equation with real coefficients.

The **Proficient** student is able to solve quadratic equations with real coefficients that have complex solutions.
The **Basic** student is able to solve quadratic equations with real coefficients that have pure imaginary solutions.
The **Below Basic** student may be able to determine which graphs have real solutions and which graphs have complex solutions when given a series of graphical representations of quadratic equations with real coefficients.

**N.CN.F.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.**

(+) In addition to Proficient, the **Advanced** student is able to:

- Explore patterns in polynomial identities that are expressed with complex numbers. **OR**
- Compare and contrast how polynomial identities are expressed in the real and the complex number system.

(+) The **Proficient** student is able to extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.

(+) The **Basic** student is able to match equivalent forms of polynomial identities expressed with complex numbers. For example, $x^2 + 4$ as $(x + 2i)(x - 2i)$.

(+) The **Below Basic** student does not meet the basic performance level.

**N.CN.F.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.**

(+) In addition to Proficient, the **Advanced** student is able to explore the solutions to quadratic polynomials with complex coefficients to identify patterns supporting the Fundamental Theorem of Algebra.

(+) The **Proficient** student is able to know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

(+) The **Basic** student is able to solve a given quadratic polynomial with real coefficients and discuss how the Fundamental Theorem of Algebra is validated.

(+) The **Below Basic** student does not meet the basic performance level.

## VECTOR AND MATRIX QUANTITIES

### Represent and model with vector quantities.

**N.VM.G.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $v, |v|, ||v||, \boldsymbol{v}$).**

(+) In addition to Proficient, the **Advanced** student is able to create real-world examples in different units that are represented as vectors. [This is identified as having a cross-curricular connection to physics.]

(+) The **Proficient** student is able to recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $v, |v|, ||v||, v$).

(+) The **Basic** student is able to identify quantities that could be represented with a vector when given a real-world example (e.g., I drove 10 mph versus I drove west at 10 mph).

(+) The **Below Basic** student does not meet the basic performance level.

**N.VM.G.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.**

(+) In addition to Proficient, the **Advanced** student is able to model a real-world situation where subtracting the initial and terminal points of a vector would be applied.

(+) The **Proficient** student is able to find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

(+) The **Basic** student is able to identify the initial and terminal points when a vector is represented graphically.

(+) The **Below Basic** student does not meet the basic performance level.

**N.VM.G.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.**

(+) In addition to Proficient, the **Advanced** student is able to create and solve real-world problems involving velocity and other quantities that can be represented by vectors.

(+) The **Proficient** student is able to solve problems involving velocity and other quantities that can be represented by vectors.

(+) The **Basic** student is able to solve problems involving velocity that can be represented by vectors.

(+) The **Below Basic** student does not meet the basic performance level.

## Perform operations on vectors.

**N.VM.H.4 (+) Add and subtract vectors.**

**N.VM.H.4A** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

**N.VM.H.4B** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

**N.VM.H.4C** Understand vector subtraction $v - w$ as $v + (-w)$, where $(-w)$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

(+) In addition to Proficient, the **Advanced** student is able to create a real-world problem that requires addition or subtraction of two or more vectors, find the resultant vector, and interpret the magnitude and direction of the resultant vector.

(+) The **Proficient** student is able to add and subtract vectors.

A. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

B. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

C. Understand vector subtraction $v - w$ as $v + (-w)$, where $(-w)$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

(+) The **Basic** student is able to add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

(+) The **Below Basic** student does not meet the basic performance level.

**N.VM.H.5 (+) Multiply a vector by a scalar.**

**N.VM.H.5A** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise.

**N.VM.H.5B** Compute the magnitude of a scalar multiple $cv$ using $||cv|| = |c|v$. Compute the direction of $cv$ knowing that when $|c|v \neq 0$, the direction of $cv$ is either along $v$ (for $c > 0$) or against $v$ (for $c < 0$).

(+) In addition to Proficient, the **Advanced** student is able to Create a real-world problem that requires scalar multiplication, find the resultant vector, and interpret the magnitude and direction of the resultant vector.

(+) The **Proficient** student is able to multiply a vector by a scalar.

A. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise.

B. Compute the magnitude of a scalar multiple $cv$ using $||cv|| = |c|v$. Compute the direction of $cv$ knowing that when $|c|v \neq 0$, the direction of $cv$ is either along $v$ (for $c > 0$) or against $v$ (for $c < 0$).

(+) The **Basic** student is able to represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise.
(+) The **Below Basic** student does not meet the basic performance level.

## Perform operations on matrices and use matrices in applications.

**N.VM.I.6 (+) Use matrices to represent and manipulate data.**

(+) In addition to Proficient, the **Advanced** student is able to using data from a real-world situation, create a matrix, analyze the situation, and make decisions.
(+) The **Proficient** student is able to use matrices to represent and manipulate data.
(+) The **Basic** student is able to state what the data represents for elements in a given matrix.
(+) The **Below Basic** student does not meet the basic performance level.

**N.VM.I.7 (+) Multiply matrices by scalars to produce new matrices.**

(+) In addition to Proficient, the **Advanced** student is able to, in a real-world situation involving scalar multiplication, place data in a matrix, identify and interpret the scalar, and interpret the results.
(+) The **Proficient** student is able to multiply matrices by scalars to produce new matrices.
(+) The **Basic** student is able to multiply matrices by integer scalars to produce new matrices.
(+) The **Below Basic** student does not meet the basic performance level.

**N.VM.I.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.**

(+) In addition to Proficient, the **Advanced** student is able to apply addition, subtraction, and/or multiplication of matrices to real-world situations.
**Recommendation**: Allow use of technology for computations of matrices with m and n greater than 3.
(+) The **Proficient** student is able to add, subtract, and multiply matrices of appropriate dimensions.
**Recommendation**: Allow use of technology for computations of matrices with m and n greater than 3.
(+) The **Basic** student is able to add and subtract matrices of dimensions limited to m and n less than or equal to 3.
(+) The **Below Basic** student does not meet the basic performance level.

**N.VM.I.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the Associative and Distributive Properties.**

(+) In addition to Proficient, the **Advanced** student is able to compare and contrast the Commutative, Associative, and Distributive Properties for the addition, subtraction, and multiplication of non-square matrices versus the addition, subtraction, and multiplication of real-numbers.
(+) The **Proficient** student is able to understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the Associative and Distributive Properties.
(+) The **Basic** student is able to calculate $AB$, $BA$, $(AB)C$, $A(BC)$, $A(B+C)$, $AB+AC$ and determine which expressions are equivalent when given $2 \times 2$ matrices A, B, and C.
(+) The **Below Basic** student does not meet the basic performance level.

**N.VM.I.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.**

(+) In addition to Proficient, the **Advanced** student is able to calculate the determinant, $|A|$, of a square matrix. Find the inverse, $A^{-1}$, using the determinant, $|A|$. Show that $A(A^{-1})$ is equal to the identity matrix (I).
**Recommendation**: Allow use of technology for computations of matrices with m greater than or equal to 3.
(+) The **Proficient** student is able to understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
(+) The **Basic** student is able to calculate $AI$, $IA$, $A + 0$, $0 + A$, and determine which expressions are equivalent to $A$ when given $2 \times 2$ matrices $A$, the zero matrix $(0)$, and the identity matrix $(I)$.
(+) The **Below Basic** student does not meet the basic performance level.

**N.VM.I.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.**

(+) In addition to Proficient, the **Advanced** student is able to create a polygon on a coordinate grid, develop the appropriate matrix to represent the polygon, use vectors to translate the shape, and graph the translated polygon on the same coordinate grid.
**Recommendation**: Use technology for polygons with more than 3 sides.
(+) The **Proficient** student is able to multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
(+) The **Basic** student is able to multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector.
(+) The **Below Basic** student does not meet the basic performance level.

**N.VM.I.12 (+) Work with 2 X 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.**

(+) In addition to Proficient, the **Advanced** student is able to create a polygon on a coordinate grid, develop the appropriate matrix to represent the polygon, use vectors to transform the shape, graph the transformed polygon on the same coordinate grid, find the area for each polygon, and compare the areas. **Recommendation**: Use technology for polygons with more than 3 sides.
(+) The **Proficient** student is able to work with $2 \times 2$ matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
(+) The **Basic** student is able to identify the transformation that is created by a given $2 \times 2$ transformation matrix.
(+) The **Below Basic** student does not meet the basic performance level.

## SEEING STRUCTURE IN EXPRESSIONS

### Interpret the structure of expressions.

**A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.**

**A.SSE.A.1A** Interpret parts of an expression, such as terms, factors, and coefficients.

**A.SSE.A.1B** Interpret complicated expressions by viewing one or more of their parts as a single entity.

In addition to Proficient, the **Advanced** student is able to:

- Interpret the effect of changes made to a term, a factor, or a coefficient in an expression. **OR**
- Compose complicated expressions from simpler ones and decompose complicated expressions into simpler ones.

The **Proficient** student is able to interpret expressions that represent a quantity in terms of its context.

A. Interpret parts of an expression, such as terms, factors, and coefficients.

B. Interpret complicated expressions by viewing one or more of their parts as a single entity.

The **Basic** student is able to interpret expressions that represent a quantity in terms of its context by interpreting parts of an expression, such as terms, factors, and coefficients.

The **Below Basic** student may be able to interpret expressions that represent a quantity in terms of its context by interpreting parts of an expression, such as terms, factors, or coefficients.

**A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.**

In addition to Proficient, the **Advanced** student is able to explain why various forms of equivalent expressions are more advantageous in a given situation.

The **Proficient** student is able to use the structure of an expression to identify ways to rewrite it.

The **Basic** student is able to rewrite an expression into another equivalent form.

The **Below Basic** student may be able to determine if two given expressions are equivalent.

### Write expressions in equivalent forms to solve problems.

**A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.**

**A.SSE.B.3A** Factor a quadratic expression to reveal the zeros of the function it defines.

**A.SSE.B.3B** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

**A.SSE.B.3C** Use the properties of exponents to transform expressions for exponential functions. Apply the concepts of decimal and scientific notation to solve real-world and mathematical problems.

i. Multiply and divide numbers expressed in both decimal and scientific notation.

ii. Add and subtract numbers in scientific notation with the same integer exponent.

In addition to Proficient, the **Advanced** student is able to choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A. Factor a quadratic expression to reveal the zeros of the function it defines and interpret the results in a real-world context.

B. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines and interpret the results in a real-world context.

The **Proficient** student is able to choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A. Factor a quadratic expression to reveal the zeros of the function it defines.

B. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
C. Use the properties of exponents to transform expressions for exponential functions. Apply the concepts of decimal and scientific notation to solve real-world and mathematical problems.
I. Multiply and divide numbers expressed in both decimal and scientific notation.
II. Add and subtract numbers in scientific notation with the same integer exponent.

The **Basic** student is able to choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- Factor a quadratic expression to reveal the zeros of the function it defines. **AND**
- Use the properties of exponents to transform expressions for exponential functions.

The **Below Basic** student may be able to choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- Factor a quadratic expression to reveal the zeros of the function it defines. **OR**
- Use the properties of exponents to transform expressions for exponential functions.

**A.SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.**

In addition to Proficient, the **Advanced** student is able to:

- Use the formula for the sum of a finite geometric series (when the common ratio is not 1) to solve real-world problems. **OR**
- Write the series in proper summation notation.

The **Proficient** student is able to derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

The **Basic** student is able to use the given formula for the sum of a finite geometric series (when the common ratio is not 1) to solve problems.

The **Below Basic** student may be able to:

- Write a geometric sequence as a finite geometric series and calculate its sum. **AND**
- Identify the common ratio and initial term of a finite geometric series (when the common ratio is not 1).

**Assessment Boundary:** n is less than 10 in the geometric sequence.

## ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS

### Perform arithmetic operations on polynomials.

**A.APR.C.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.**

In addition to Proficient, the **Advanced** student is able to:

- Rewrite a polynomial expression involving multiplication, and addition or subtraction, into an equivalent polynomial expression in standard form. **OR**
- Generalize a pattern when adding, subtracting, and multiplying polynomials of a varying number of terms.

The **Proficient** student is able to understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

The **Basic** student is able to add, subtract, and multiply polynomials.

The **Below Basic** student may be able to add, subtract, and multiply binomials.

## Understand the relationship between zeros and factors of polynomial.

**A.APR.D.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by $(x - a)$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.**

In addition to Proficient, the **Advanced** student is able to find a missing coefficient $m$, where $m$ is a real number, of a polynomial $p(x)$, when given $(x - a)$ is a factor of $p(x)$ or when given $p(a)$.

The **Proficient** student is able to know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $(x - a)$ is $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

**Assessment Boundary:** degree of $p(x) = 3$

The **Basic** student is able to evaluate $p(a)$ and compare it to the remainder of $p(x) / (x - a)$. Explain the significance of the remainder.

**Assessment Boundary:** degree of $p(x) = 3$

The **Below Basic** student may be able to evaluate $p(a)$ and compare it to the remainder of $p(x) / (x - a)$. Explain the significance of the remainder.

**Assessment Boundary:** degree of $p(x) = 2$

**A.APR.D.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.**

In addition to Proficient, the **Advanced** student is able to given a graph of a polynomial function with integer x-intercepts, write the general form of the polynomial in standard form, understanding that the polynomial could have a stretch or compression.

The **Proficient** student is able to identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

The **Basic** student is able to identify zeros of polynomials when suitable factorizations are available, and locate the zeros on a coordinate plane.

The **Below Basic** student may be able to match the polynomial to its factored form and its graph.

## Use polynomial identities to solve problems.

**A.APR.E.4 Prove polynomial identities and use them to describe numerical relationships.**

In addition to Proficient, the **Advanced** student is able to use the structure of a polynomial identity to give real-world contextual meaning to the identity (e.g., completing the square of a quadratic function to highlight the maximum or minimum, factoring a polynomial to highlight the zeros, or factoring a trinomial to highlight base times width).

The **Proficient** student is able to prove polynomial identities and use them to describe numerical relationships.

The **Basic** student is able to evaluate each polynomial expression for given values, compare the results, and make a general statement concerning the polynomials as identities.

The **Below Basic** student may be able to given two polynomial identities, evaluate the polynomials at a given value to demonstrate that the polynomials are identities.

**A.APR.E.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$, where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal's Triangle.**

(+) In addition to Proficient, the **Advanced** student is able to find specified terms in the expansion of $(x + y)^n$ by applying properties of the binomial theorem.
(+) The **Proficient** student is able to know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$, where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal's Triangle.
(+) The **Basic** student is able to expand $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$, where $x$ and $y$ are any numbers, using Pascal's Triangle.
(+) The **Below Basic** student does not meet the basic performance level.

## Rewrite rational expressions.

**A.APR.F.6 Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ using inspection, long division, or, for the more complicated examples, a computer algebra system. (i.e. rewriting a rational expression as the quotient plus the remainder over divisor).**

In addition to Proficient, the **Advanced** student is able to, given $\frac{a(x)}{b(x)} = q(x) + \frac{r(x)}{b(x)}$:

- Identify the missing coefficient from a polynomial $a(x)$ when given $b(x)$, $q(x)$, and $r(x)$. **OR**
- Generalize the patterns obtained by dividing various degrees of polynomials.

The **Proficient** student is able to rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ using inspection, long division, or, for the more complicated examples, a computer algebra system. (i.e. rewriting a rational expression as the quotient plus the remainder over divisor).

The **Basic** student is able to rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ using inspection or long division where $a(x)$ is a quadratic and $b(x)$ is linear.
The **Below Basic** student may be able to:

- Match a rational expression in the form $\frac{a(x)}{b(x)}$ to its equivalent form $q(x) + \frac{r(x)}{b(x)}$. **OR**
- Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ using inspection or long division where $a(x)$ is a quadratic and $b(x)$ is linear with assistance.

**A.APR.F.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.**

(+) In addition to Proficient, the **Advanced** student is able to justify that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.
(+) The **Proficient** student is able to understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
(+) The **Basic** student is able to add, subtract, multiply, and divide rational expressions.
(+) The **Below Basic** student does not meet the basic performance level.

## CREATING EQUATIONS

### Create equations that describe numbers or relationships.

**A.CED.G.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**

In addition to Proficient, the **Advanced** student is able to create equations and inequalities in one variable and use them to solve problems. Include compound inequalities arising from problems. Use interval notation to represent inequalities.
The **Proficient** student is able to create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
The **Basic** student is able to create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, and simple exponential functions.
The **Below Basic** student may be able to create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions.

**A.CED.G.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.**

In addition to Proficient, the **Advanced** student is able to:

- Interpret the relationships between the graph and its corresponding equation in real-world contexts. **OR**
- Graph equations of the form $x + y + z = c$ or ordered triples on the $xyz$-plane. **OR**
- Graph equations from a real-world context of the form $y = ab^x$ where $a$ is a real number and $b$ is greater than 0.

The **Proficient** student is able to create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
The **Basic** student is able to:

- Create equations in two or more variables to represent relationships between quantities. **OR**
- Graph equations on coordinate axes with labels and scales.

The **Below Basic** student may be able to match a graph to its equation in two or more variables.

**A.CED.G.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.**

In addition to Proficient, the **Advanced** student is able to examine and explain constraints and solutions to systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
The **Proficient** student is able to represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
The **Basic** student is able to:

- Interpret solutions as viable or non-viable options in a modeling context. **AND**
- Represent constraints by equations or inequalities, **or** by systems of equations and/or inequalities.

The **Below Basic** student may be able to:

- Interpret solutions as viable or non-viable options in a modeling context. **AND**
- Represent constraints by equations or inequalities.

**A.CED.G.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.**

In addition to Proficient, the **Advanced** student is able to choose and explain reasoning for highlighting the quantity of interest, rearrange the formula, use the rearranged formula to evaluate, and interpret the answer in a real-world context.
The **Proficient** student is able to rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
The **Basic** student is able to rearrange formulas to highlight a quantity of interest in two steps, using the same reasoning as in solving equations.
The **Below Basic** student may be able to rearrange formulas to highlight a quantity of interest in one step, using the same reasoning as in solving equations.

## REASONING WITH EQUATIONS AND INEQUALITIES

### Understand solving equations as a process of reasoning and explain the reasoning.

**A.REI.H.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.**

In addition to Proficient, the **Advanced** student is able to critique the solution and justification of self and others in the steps in solving linear equations.
The **Proficient** student is able to explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
The **Basic** student is able to informally describe the steps in solving multi-step linear equations.
The **Below Basic** student may be able to match steps to justifications when provided with the steps for solving multi-step linear equations.

**A.REI.H.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.**

In addition to Proficient, the **Advanced** student is able to critique the solutions of self and others for simple rational and radical equations in one variable.
The **Proficient** student is able to solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
The **Basic** student is able to solve simple rational and radical equations in one variable with no extraneous solutions, and when given equations with extraneous solution(s) demonstrate why the given solution(s) is/are not viable.
The **Below Basic** student may be able to solve simple rational and radical equations in one variable with no extraneous solutions.

### Solve equations and inequalities in one variable.

**A.REI.I.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.**

In addition to Proficient, the **Advanced** student is able to:

- Critique the solutions of self and others for linear inequalities in one variable, including equations with coefficients represented by letters. **OR**
- Demonstrate the solution of a linear equation or inequality in multiple ways (e.g., graphically, set notation, interval notation). **OR**
- Create and solve a real-world linear equation or inequality in one variable, determine its viable domain and solution set.

The **Proficient** student is able to solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
The **Basic** student is able to solve linear equations and inequalities in one variable, limited to numerical coefficients.
The **Below Basic** student may be able to solve linear equations and inequalities in one variable, limited to numerical coefficients, where the variable is on only one side of the equal or inequality sign.

**A.REI.I.4 Solve quadratic equations in one variable.**

**A.REI.I.4A** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions.

**A.REI.I.4B** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

**A.REI.I.4C** (+) Derive the quadratic formula from the general form of a quadratic equation.

In addition to Proficient, the **Advanced** student is able to:

- Critique different methods used by self and others for solving quadratic equations in one variable. **OR**
- Create and solve a real-world quadratic equation in one variable, determine its viable domain and solution. **OR**
- Explain the purpose for the method chosen to solve the quadratic equation in a real-world situation.

The **Proficient** student is able to solve quadratic equations in one variable.

A. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions.

B. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

C. (+) Derive the quadratic formula from the general form of a quadratic equation.

The **Basic** student is able to solve quadratic equations in one variable by inspection (e.g., for $x^2 = 49$), taking square roots, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

The **Below Basic** student may be able to solve quadratic equations in one variable, using the quadratic formula. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

## Solve systems of equations.

**A.REI.J.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other, produces a system with the same solutions.**

In addition to Proficient, the **Advanced** student is able to:

- Write a different system of two equations in two variables with the same solution as a given system of two equations in two variables. **OR**
- Create and solve, if possible, a system of two equations in two variables for a real-world context (include systems with one solution, infinitely many solutions, or no solution).

The **Proficient** student is able to prove that given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other, produces a system with the same solutions.

The **Basic** student is able to solve a system of two equations in two variables with different coefficients resulting in one solution using the elimination method.

The **Below Basic** student may be able to solve a system of two equations in two variables with equal or opposite coefficients resulting in one solution using the elimination method.

**A.REI.J.6 Estimate solutions graphically and determine algebraic solutions to linear systems, focusing on pairs of linear equations in two variables.**

In addition to Proficient, the **Advanced** student is able to approximate solutions to a system of linear equations for a real-world situation using a table and graph, then verify the solution algebraically and discuss the viability of the solution in the context of the problem.
The **Proficient** student is able to estimate solutions graphically and determine algebraic solutions to linear systems, focusing on pairs of linear equations in two variables.
The **Basic** student is able to estimate solutions to linear systems graphically or determine algebraic solutions to linear systems, focusing on pairs of linear equations in two variables.
The **Below Basic** student may be able to test a solution to the system in both original equations (graphically or algebraically).

**A.REI.J.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.**

In addition to Proficient, the **Advanced** student is able to explore algebraically, graphically, and tabularly the number and type of the solutions of one linear and one quadratic or two quadratics and describe findings.
The **Proficient** student is able to solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
The **Basic** student is able to solve a simple system consisting of a linear equation written in slope-intercept form and a quadratic equation written in standard form in two variables algebraically.
The **Below Basic** student may be able to solve a simple system consisting of a linear equation written in slope-intercept form and a quadratic equation written in standard form in two variables graphically.

**A.REI.J.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.**

(+) In addition to Proficient, the **Advanced** student is able to represent a real-world situation using a system of linear equations as a single matrix equation in a vector variable.
(+) The **Proficient** student is able to represent a system of linear equations as a single matrix equation in a vector variable.
(+) The **Basic** student is able to identify a system of linear equations given a matrix equation in a vector variable.
(+) The **Below Basic** student does not meet the basic performance level.

**A.REI.J.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension $3 \times 3$ or greater).**

(+) In addition to Proficient, the **Advanced** student is able to find the inverse of a matrix if it exists and use it to solve a real-world problem involving systems of linear equations (using technology for matrices of dimension $3 \times 3$ or greater).
(+) The **Proficient** student is able to find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension $3 \times 3$ or greater).
(+) The **Basic** student is able to solve systems of linear equations written as a single matrix equation in a vector variable when given the inverse of a matrix (using technology for matrices of dimension $3 \times 3$ or greater).
(+) The **Below Basic** student does not meet the basic performance level.

## Represent and solve equations and inequalities graphically.

**A.REI.K.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.**

In addition to Proficient, the **Advanced** student is able to create and graph two equations of different degrees that pass through the same two specific points. Describe the relationship between the solution sets for each equation and the solution for the system.

The **Proficient** student is able to understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane (i.e., connect algebraic and graphical representations of an equation in two variables).

The **Basic** student is able to create and verify a set of ordered pairs that lie on the graph when given an equation.

The **Below Basic** student may be able to determine which ordered pairs do or do not lie on the graph of the equation when given an equation and a set of ordered pairs.

**A.REI.K.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.**

In addition to Proficient, the **Advanced** student is able to, given a table of ordered pairs and graph for both a linear $y = f(x)$ and quadratic $y = g(x)$ where the intersection is a non-integer coordinate pair, describe a method to find the equations and solutions matching the depicted graphs and tables. Find the solutions using the generated equations. Describe the accuracy of the solution algebraically and graphically. Describe how to improve the method.

The **Proficient** student is able to explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

The **Basic** student is able to explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions or make tables of values. Include cases where $f(x)$ and/or $g(x)$ are linear, quadratic, absolute value, and exponential.

The **Below Basic** student may be able to explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions or make tables of values. Include cases where $f(x)$ and/or $g(x)$ are linear, quadratic, and exponential.

**A.REI.K.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.**

In addition to Proficient, the **Advanced** student is able to:

- Write a system of linear inequalities that includes a set of specified points in a region and explain the choice of strict inequality or non-strict inequality notation. **OR**
- Create a system of linear inequalities that will optimize the solution when given a real-life scenario. **OR**
- Write the inequalities that describe the boundaries of that scenario and discuss the optimal solution when given a graphical representation of a real-life scenario.

The **Proficient** student is able to graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

The **Basic** student is able to graph the solution to a linear inequality in two variables as a half-plane (excluding the boundary in the case of strict inequality).

The **Below Basic** student may be able to match the solution to a linear inequality in two variables to its graph half-plane (excluding the boundary in the case of strict inequality).

## INTERPRETING FUNCTIONS

### Understand the concept of a function and use function notation.

**F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.**

In addition to Proficient, the **Advanced** student is able to use multiple representations to generalize ways to define functions and non-functions.
The **Proficient** student is able to demonstrate that a function's domain is assigned to exactly one element of the range in equations, tables, graphs, and context.
The **Basic** student is able to demonstrate that a function's domain is assigned to exactly one element of the range in equations, tables, and graphs.
The **Below Basic** student may be able to demonstrate that a function's domain is assigned to exactly one element of the range in tables and graphs.

**F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.**

In addition to Proficient, the **Advanced** student is able to create context from a given domain and range and use function notation to write an equation, graph the function, and draw a picture that models the context.
The **Proficient** student is able to use function notation, evaluate functions for inputs in their domain, and interpret statements that use function notation in terms of a context.
The **Basic** student is able to use function notation and evaluate functions for inputs in their domain.
The **Below Basic** student may be able to evaluate equations for specific values and match the equation to function notation.

**F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.**

In addition to Proficient, the **Advanced** student is able to write the explicit and recursive forms of a sequence describing linear and exponential situations. Express the sequence graphically and in a table.
The **Proficient** student is able to recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
The **Basic** student is able to write an informal recursive description of a sequence.
The **Below Basic** student may be able to write the first n terms when given an informal recursive description of a sequence.

### Interpret functions that arise in applications in terms of the context.

**F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.**

In addition to Proficient, the **Advanced** student is able to graph and label the key features of the function and write the function in function notation when given key features from a linear, exponential, or quadratic context.
The **Proficient** student is able to, for a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

The **Basic** student is able to, for linear, quadratic, and exponential functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, maximums and minimums; symmetries; end behavior.
The **Below Basic** student may be able to, for linear, quadratic, and exponential functions that model a relationship between two quantities, interpret key features of graphs in terms of the quantities, and sketch graphs showing key features given the function. Key features include: intercepts; intervals where the function is increasing, decreasing, maximums and minimums; symmetries; end behavior.

**F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.**

In addition to Proficient, the **Advanced** student is able to:

- Write and graph a function for a given context where the domain meets given parameters. Express the domain of the function using interval and/or set notation as appropriate. **OR**
- Given the graph of a function, write its domain and range using interval and/or set notation as appropriate.

The **Proficient** student is able to relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
The **Basic** student is able to match the domain of a function to its graph and explain why the selected domain is applicable to the situation.
The **Below Basic** student may be able to match the domain of a function to its graph.

**F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.**

In addition to Proficient, the **Advanced** student is able to:

- Analyze the difference between the rates of change of different types of functions. **OR**
- Compare average rates of change over different intervals of the same function. **OR**
- Generalize how the average rate of change differs between different function types.

The **Proficient** student is able to calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
The **Basic** student is able to calculate and interpret the average rate of change of a linear, exponential, or quadratic function over a specified interval presented as a graph, an equation, or a table.
The **Below Basic** student may be able to calculate and interpret the average rate of change of a linear, exponential, or quadratic function over a specified interval presented as a graph.

## Analyze functions using different representations.

**F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.**

**F.IF.C.7A** Graph linear and quadratic functions and show intercepts, maxima, and minima.
**F.IF.C.7B** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
**F.IF.C.7C** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
**F.IF.C.7D** (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
**F.IF.C.7E** Graph exponential and logarithmic functions, showing intercepts and end behavior.
**F.IF.C.7F** (+) Graph trigonometric functions, showing period, midline, and amplitude.

In addition to Proficient, the **Advanced** student is able to create different representations of linear, quadratic, and exponential functions when given one of the following representations: graphical, tabular, or algebraic. Compare and contrast all three function types identifying key features while referencing the representations.

The **Proficient** student is able to graph linear, quadratic, and exponential functions expressed symbolically and show appropriate key features of the graph showing intercepts, maxima, and minima, and end behavior.

A. Graph linear and quadratic functions and show intercepts, maxima, and minima.
B. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
C. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
E. Graph exponential and logarithmic functions, showing intercepts and end behavior
D. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
F. (+) Graph trigonometric functions, showing period, midline, and amplitude.

The **Basic** student is able to identify appropriate key features of linear, quadratic, and exponential functions from a graph showing intercepts (linear, quadratic, and exponential), maximum or minimum (quadratic), and end behavior (linear, quadratic, and end behavior).

A. Graph linear and quadratic functions and show intercepts (linear and quadratic) and maxima or minima (quadratic).
B. Graph square root, cube root, and absolute value functions.
C. Graph polynomial functions, identifying zeros and showing end behavior.
E. Graph exponential and logarithmic functions, showing intercepts and end behavior.
D. (+) Graph rational functions, identifying zeros and asymptotes, and showing end behavior.
F. (+) Graph trigonometric functions, showing period, midline, and amplitude.

The **Below Basic** student may be able to match descriptions of key features of linear, quadratic, and exponential functions to the appropriate parts of the graph including intercepts (linear, quadratic, and exponential), maximum or minimum (quadratic), and end behavior (linear, quadratic, and end behavior).

A. For linear and quadratic functions, identify intercepts (linear and quadratic) and maxima or minima (quadratic).
B. For square root, cube root, and absolute value functions identify intercepts, symmetry, and end behavior.
C. For polynomial functions, identify intercepts and end behavior.
E. For exponential and logarithmic functions, identify intercepts and end behavior.
D. (+) For rational functions, identify zeros, asymptotes, and end behavior.
F. (+) For trigonometric functions with no vertical shift, identify amplitude and period using a set of x-intercepts.

**F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.**

**F.IF.C.8A** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

**F.IF.C.8B** Use the properties of exponents to interpret expressions for exponential functions.

In addition to Proficient, the **Advanced** student is able to write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

A. Compare and contrast factored, vertex, and standard form to discuss the appropriate form of various properties (zeros, extreme values, and symmetry). Write a quadratic function in different forms to reveal the appropriate properties and compare those properties to the function's graph or table of values in a real-world context.

B. Write an exponential function from a real-world context and use the properties of exponents to interpret the expression in the context of the situation.

The **Proficient** student is able to write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

A. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

B. Use the properties of exponents to interpret expressions for exponential functions.

The **Basic** student is able to write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

A. Use the process of factoring to show zeros. Compare standard and vertex forms of a quadratic function to show extreme values and symmetry of the graph. Interpret these in terms of a context.

B. Use the properties of exponents to evaluate expressions for exponential functions.

The **Below Basic** student may be able to write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

A. Match the factored form to its graph to identify zeros. Match vertex form of a quadratic function to its graph to show extreme values and symmetry of the graph.

B. Match properties of exponential functions to the appropriate part of the exponential expression.

**F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).**

In addition to Proficient, the **Advanced** student is able to create different representations of two functions (algebraically, graphically, numerically in tables, or by verbal description) from a given representation. Compare and contrast properties of those functions, specifically highlighting what each representation reveals.

The **Proficient** student is able to compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

The **Basic** student is able to compare properties of two functions when given only two different representations at a time (algebraically, graphically, numerically in tables, or by verbal descriptions).

The **Below Basic** student may be able to match properties of two functions when given only two different representations at a time (algebraically, graphically, numerically in tables, or by verbal descriptions).

# BUILDING FUNCTIONS

## Build a function that models a relationship between two quantities.

**F.BF.D.1 Write a function that describes a relationship between two quantities.**

**F.BF.D.1A** Determine an explicit expression, a recursive process, or steps for calculation from a context.

**F.BF.D.1B** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

**F.BF.D.1C** (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

In addition to Proficient, the **Advanced** student is able to write a function that describes a relationship between two quantities.

A. Write two or more explicit expressions to express a single sequence shown pictorially and compare their features.
B. Graphically and tabularly combine standard function types using arithmetic operations.
C. (+) Determine which field properties hold under compositions. Determine under what conditions the Commutative Property holds for composition.

The **Proficient** student is able to write a function that describes a relationship between two quantities.

A. Determine an explicit expression, a recursive process, or steps for calculation from a context.
B. Combine standard function types using arithmetic operations.
C. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

The **Basic** student is able to write a function that describes a relationship between two quantities.

A. Determine an explicit expression, a recursive process, or steps for calculation from a context for linear and exponential relationships.
B. Combine standard function types using arithmetic operations for addition, subtraction, and multiplication of binomials.
C. (+) Compose functions numerically and graphically, and interpret the solution in context.

The **Below Basic** student may be able to write a function that describes a relationship between two quantities.

A. Determine an explicit expression, a recursive process, or steps for calculation from a context for linear relationships.
B. Combine standard function types using arithmetic operations for addition, subtraction, and multiplication of linear binomials.
C. (+) Compose functions numerically, and interpret the solution in context.

**F.BF.D.2 (+) Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.**

(+) In addition to Proficient, the **Advanced** student is able to:

- Compare the graphical representation, tabular representation, explicit and recursive formulas, and contextual representation for arithmetic and geometric sequences. Draw parallels to linear and exponential functions, respectively. **AND/OR**
- Describe a real-world situation that can be modeled linearly or exponentially and develop the explicit or recursive formulas to model the situation.

(+) The **Proficient** student is able to write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

(+) The **Basic** student is able to write arithmetic and geometric sequences both recursively and with an explicit formula given a modeling situation.

(+) The **Below Basic** student may be able to write an explicit or recursive formula for the model of an arithmetic and geometric sequence given the other formula.

## Build new functions from existing functions.

**F.BF.E.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.**

In addition to Proficient, the **Advanced** student is able to:

- Write the equation for a transformed parent function given the graph or verbal description of the transformations. **OR**
- Write a description of the transformations using function notation given a verbal description of the transformations of $f(x)$ or the original $f(x)$ graph and its transformed graph. **OR**
- Generalize effects of a transformation on domains and ranges, including effects on ordered pairs, when exploring all of the different types of transformations in multiple representations of $f(x)$.

The **Proficient** student is able to identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

The **Basic** student is able to identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

The **Below Basic** student may be able to match the equation to the graph showing the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); write a description of the transformation. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

**F.BF.E.4 Find inverse functions.**

**F.BF.E.4A** Write an expression for the inverse of a simple, invertible function $f(x)$. Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, as $f$ and $g$ are inverse functions, if and only if, $f(x) = y$ and $g(y) = x$, for all values of x in the domain of $f$ and all values of $y$ in the domain of $g$.

**F.BF.E.4B** (+) Verify by composition that one function is the inverse of another.

**F.BF.E.4C** (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

**F.BF.E.4D** (+) Produce an invertible function from a non-invertible function by restricting the domain.

In addition to Proficient, the **Advanced** student is able to find inverse functions.

A. Create a model of a function and its inverse from a real-world context.

B. (+) Compare the equation, table, and graph when composing two functions $f(g(x))$ and $g(f(x))$. Determine what happens to the equations, table values, and graphs of $f(g(x))$ and $g(f(x))$ when $f$ and $g$ are inverses.

C. (+) Create a table for two functions in such a way that one is invertible and the other is not. Create graphs for the functions and connect features of the graph to properties of invertible functions.

D. (+) Show the relationship between the properties of a function and its inverse (domain, range, increasing, decreasing, asymptotes, intercepts).

The **Proficient** student is able to find inverse functions.

A. Write an expression for the inverse of a simple, invertible function $f(x)$. Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, as f and g are inverse functions, if and only if, $f(x) = y$ and $g(y) = x$, for all values of $x$ in the domain of $f$ and all values of $y$ in the domain of $g$.

B. (+) Verify by composition that one function is the inverse of another.

C. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
D. (+) Produce an invertible function from a non-invertible function by restricting the domain.

The **Basic** student is able to understand inverse functions.

A. Match expressions for a function and its inverse.
B. (+) Identify the compositions that shows $f$ and $g$ are inverses.
C. (+) Identify values of an inverse function from a table, given that the function has an inverse.
D. (+) Restrict the domain of a non-invertible function to make it invertible given a graph.

The **Below Basic** student may be able to understand inverse functions.

A. Match the graph and/or table of a function to the graph and/or table of its inverse.
B. (+) Show that one function may be the inverse of another by using numerical composition.
C. (+) Create a table for an inverse of a function, given a function table.
D. (+) Determine if a function will have an inverse from a given graph.

**F.BF.E.5 (+) Build new functions from existing functions. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.**

(+) In addition to Proficient, the **Advanced** student is able to solve real-world problems involving logarithms and exponents.

(+) The **Proficient** student is able to understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

(+) The **Basic** student is able to:

- Given an expression written in logarithmic form, write the equivalent expression in exponential form. **AND**
- Given an expression in exponential form, write the equivalent expression in logarithmic form.

(+) The **Below Basic** student does not meet the basic performance level.

## LINEAR, QUADRATIC, AND EXPONENTIAL MODELS

### Construct and compare linear, quadratic, and exponential models and solve problems.

**F.LE.F.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.**

**F.LE.F.1A** Verify that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

**F.LE.F.1B** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

**F.LE.F.1C** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

In addition to Proficient, the **Advanced** student is able to distinguish between situations that can be modeled with linear functions and with exponential functions.

A. Write an equation, sketch a graph, and create a table of values given linear and exponential real-world contexts and explain how the growth changes.
B. Predict and compare values along a continuum of linear and exponential functions in real-world contexts.
C. Compare and contrast the rates of change of linear and exponential functions. Write a generalization about rates of change.

The **Proficient** student is able to distinguish between situations that can be modeled with linear functions and with exponential functions.

A. Verify that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

B. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
C. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

The **Basic** student is able to distinguish between situations that can be modeled with linear functions and with exponential functions.

A. Demonstrate that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals from a table of values.
B. Match situations in which one quantity changes at a constant rate per unit interval relative to another.
C. Match situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

The **Below Basic** student may be able to distinguish between situations that can be modeled with linear functions and with exponential functions.

A. Informally show that linear functions grow by equal differences over equal intervals by calculating rate of change for two sets of order pairs. Informally show that exponential functions grow by equal factors over equal intervals by showing multiplicative growth for two sets of ordered pairs.
B. Identify graphs in which one quantity changes at a constant rate per unit interval relative to another.
C. Identify graphs in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

**F.LE.F.2 Construct linear and exponential functions using a graph, a description of a relationship, or two input-output pairs (include reading these from a table).**

In addition to Proficient, the **Advanced** student is able to:

- Explain how exponential and linear function values differ as $x$ approaches positive or negative infinity. **AND**
- Relate algebraic representations of linear and exponential functions to the explicit and recursive forms of arithmetic and geometric sequences, respectively. **AND/OR**
- Create a real-world scenario and develop linear or exponential tables or graphs representing the scenario.

The **Proficient** student is able to construct linear and exponential functions using a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

The **Basic** student is able to construct linear and exponential functions using two of the following representations: a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

The **Below Basic** student may be able to match linear and exponential functions to their graph, description of a relationship, and two input-output pairs (include reading these from a table).

**F.LE.F.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.**

In addition to Proficient, the **Advanced** student is able to use an algebraic argument or informal proof to show that an increasing exponential function eventually exceeds an increasing linear function.

The **Proficient** student is able to observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

The **Basic** student is able to identify which function eventually exceeds the others when given the graphs and tables of increasing linear, quadratic, and exponential functions.

The **Below Basic** student may be able to compare the output values for increasing $x$-values of increasing linear, quadratic, and exponential functions to determine which function has the greatest output value.

**F.LE.F.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a, c,$ and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.**

In addition to Proficient, the **Advanced** student is able to construct an exponential model using given contextual data. Predict the input from a graph or table then use logarithms to find the exact solution of an independent value from the context, given a dependent value. Compare and discuss the exact solution to a graphical or tabular solution.

The **Proficient** student is able to, for exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a, c,$ and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.

The **Basic** student is able to, for exponential models, express as a logarithm the solution to $ab^{t} = d$ where $a$ and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.

The **Below Basic** student may be able to, for exponential models, express as a logarithm the solution to $b^{t} = d$ where $d$ is a real number and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.

### Interpret expressions for functions in terms of the situation they model.

**F.LE.G.5 Interpret the parameters in a linear or exponential function in terms of a context.**

In addition to Proficient, the **Advanced** student is able to model real-world scenarios with linear and exponential functions with appropriate parameters.

The **Proficient** student is able to interpret the parameters in a linear or exponential function in terms of a context.

The **Basic** student is able to identify appropriate parameters for linear or exponential functions in terms of a context.

The **Below Basic** student may be able to match parameters for linear or exponential functions to the appropriate parts of the graph.

## TRIGONOMETRIC FUNCTIONS

### Extend the domain of trigonometric functions using the unit circle.

**F.TF.H.1 (+) Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.**

(+) In addition to Proficient, the **Advanced** student is able to solve problems using radian measure.

(+) The **Proficient** student is able to demonstrate that radian measure of an angle is the length of the arc on the unit circle subtended by the angle.

(+) The **Basic** student is able to convert between degree and radian measures.

(+) The **Below Basic** student does not meet the basic performance level.

**F.TF.H.2 (+) Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.**

(+) In addition to Proficient, the **Advanced** student is able to explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed clockwise and counterclockwise around the unit circle.

(+) The **Proficient** student is able to explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

(+) The **Basic** student is able to explain how the unit circle in the coordinate plane enables the extension of the sine and cosine functions to the special angles (e.g., $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$), interpreted as radian measures of angles traversed counterclockwise once around the unit circle.

(+) The **Below Basic** student does not meet the basic performance level.

**F.TF.H.3 (+) Use special triangles to determine geometrically the values of sine, cosine, and tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for $x$, where $x$ is any real number.**

(+) In addition to Proficient, the **Advanced** student is able to use special triangles to determine geometrically the values of the six trigonometric functions for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$, and use the unit circle to express the values of the six trigonometric functions for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for $x$, where $x$ is any real number.

(+) The **Proficient** student is able to use special triangles to determine geometrically the values of sine, cosine, and tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for $x$, where $x$ is any real number.

(+) The **Basic** student is able to use special triangles to determine geometrically the values of sine and cosine for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$, and use the unit circle to express the values of sine and cosine for the reference angles of $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$.

(+) The **Below Basic** student does not meet the basic performance level.

**F.TF.H.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.**

(+) In addition to Proficient, the **Advanced** student is able to verify the relationships for symmetry (odd and even) of trigonometric functions holds for values of theta outside of the first quadrant of the unit circle.

(+) The **Proficient** student is able to use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

(+) The **Basic** student is able to use the unit circle to explain periodicity of trigonometric functions.

(+) The **Below Basic** student does not meet the basic performance level.

## Model periodic phenomena with trigonometric functions.

**F.TF.I.5 (+) Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.**

(+) In addition to Proficient, the **Advanced** student is able to model trigonometric functions of periodic phenomena by identifying amplitude, frequency, and midline, and writing the equation when given a data set.

(+) The **Proficient** student is able to choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

(+) The **Basic** student is able to choose trigonometric functions to model periodic phenomena with specified amplitude and midline.

(+) The **Below Basic** student does not meet the basic performance level.

**F.TF.I.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.**

(+) In addition to Proficient, the **Advanced** student is able to identify key features of an inverse trigonometric function when given a trigonometric function whose domain is always increasing or always decreasing.

(+) The **Proficient** student is able to understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

(+) The **Basic** student is able to understand that restricting the sine and cosine functions to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

(+) The **Below Basic** student does not meet the basic performance level.

**F.TF.I.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.**

(+) In addition to Proficient, the **Advanced** student is able to use inverse functions to solve trigonometric equations that arise in modeling contexts; extrapolate solutions to the model based on periodicity, and interpret them in terms of the context.
(+) The **Proficient** student is able to use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
(+) The **Basic** student is able to use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology.
(+) The **Below Basic** student does not meet the basic performance level.

### Prove and apply trigonometric identities.

**F.TF.J.8 (+) Prove the Pythagorean identity $sin^2 A + cos^2 A = 1$ and use it to find $sin A$, $cos A$, or $tan A$, given $sin A$, $cos A$, or $tan A$, and the quadrant of the angle.**

(+) In addition to Proficient, the **Advanced** student is able to prove the other two Pythagorean identities using $sin^2 A + cos^2 A = 1$.
(+) The **Proficient** student is able to prove the Pythagorean identity $sin^2 A + cos^2 A = 1$ and use it to find $sin A$, $cos A$, or $tan A$, given $sin A$, $cos A$, or $tan A$, and the quadrant of the angle.
(+) The **Basic** student is able to use the Pythagorean identity $sin^2 A + cos^2 A = 1$ to find $sin A$ or $cos A$ given $sin A$ or $cos A$ and the quadrant of the angle.
(+) The **Below Basic** student does not meet the basic performance level.

**F.TF.J.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.**

(+) In addition to Proficient, the **Advanced** student is able to use the addition formula to prove multiple-angle identities and use them to solve problems.
(+) The **Proficient** student is able to prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
(+) The **Basic** student is able to use the addition and subtraction formulas for sine and cosine to solve problems.
(+) The **Below Basic** student does not meet the basic performance level.

## CONGRUENCE

### Experiment with transformations in the plane.

**G.CO.A.1 Apply precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.**

In addition to Proficient, the **Advanced** student is able to apply and justify the use of precise definitions while synthetically and/or analytically solving problems.
The **Proficient** student is able to apply precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
The **Basic** student is able to define an angle, circle, perpendicular line, parallel line, and line segment in simple terms.
The **Below Basic** student may be able to identify an angle, circle, perpendicular line, parallel line, and line segment in simple terms given contextual and/or illustrative choices.

**G.CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).**

In addition to Proficient, the **Advanced** student is able to generalize the representations of rigid transformations by using and recognizing transformations in other areas of mathematics.

The **Proficient** student is able to represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

The **Basic** student is able to perform two of the following:

- Represent transformations in the plane using, e.g., transparencies and geometry software.
- Describe transformations as functions that take points in the plane as inputs and give other points as outputs.
- Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

The **Below Basic** student may be able to perform one of the following:

- Represent transformations in the plane using, e.g., transparencies and geometry software.
- Describe transformations as functions that take points in the plane as inputs and give other points as outputs.
- Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

**G.CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.**

In addition to Proficient, the **Advanced** student is able to utilize reflective and rotational symmetry to describe irregular polygons or algebraic functions.

The **Proficient** student is able to, given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

The **Basic** student is able to determine if some, but not all, shapes (rectangle, parallelogram, trapezoid, or regular polygon) have rotational and/or reflective symmetry.

The **Below Basic** student may be able to choose a rectangle, parallelogram, trapezoid, or regular polygon and determine if it has rotational or reflective symmetry and/or identify if a figure has been rotated or reflected when given a graph or picture.

**G.CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.**

In addition to Proficient, the **Advanced** student is able to apply the definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments to real-world situations.

The **Proficient** student is able to develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

The **Basic** student is able to develop definitions of rotations, reflections, and/or translations in terms of angles, circles, perpendicular lines, parallel lines, and/or line segments.

The **Below Basic** student may be able to identify rotations, reflections, and/or translations in terms of angles, circles, perpendicular lines, parallel lines, and/or line segments.

**G.CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.**

In addition to Proficient, the **Advanced** student is able to generate a geometric figure identifying the rotations, reflections, or translations used in its creation. Investigate to determine an efficient sequence of transformations that will carry a given figure onto another.
The **Proficient** student is able to, given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
The **Basic** student is able to sketch the image of the figure when given a geometric figure and a transformation described in words, or given a sketch, describe the transformation in writing or verbally.
The **Below Basic** student may be able to, given a figure, identify and describe the transformation performed on a given shape in simple terms.

## Understand congruence in terms of rigid motions.

**G.CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.**

In addition to Proficient, the **Advanced** student is able to use the definition of congruence, in terms of rigid motions, to construct a viable argument that two figures are congruent and/or predict the effect on a given rigid motion on an algebraic function.
The **Proficient** student is able to use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
The **Basic** student is able to recognize congruency and/or be able to predict the effect on a rigid motion on a given figure.
The **Below Basic** student may be able to determine if two figures are congruent and explain their reasoning.

**G.CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.**

In addition to Proficient, the **Advanced** student is able to construct a viable argument using the definition of congruence, in terms of rigid motions, to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent and/or use counter-examples.
The **Proficient** student is able to use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
The **Basic** student is able to recognize and identify that two triangles are congruent using rigid transformation.
The **Below Basic** student may be able to distinguish between congruent and non-congruent triangles.

**G.CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.**

In addition to Proficient, the **Advanced** student is able to explain and provide examples showing that the criteria (SSA, AAA, and SAA) do not always prove triangles congruent.
The **Proficient** student is able to explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
The **Basic** student is able to identify the criteria for triangle congruence (ASA, SAS, and/or SSS) follow from definition of congruence using rigid motions, using tools such as rulers, protractors, distance formula, etc.
The **Below Basic** student may be able to identify corresponding parts in two triangles.

## Prove geometric theorems.

**G.CO.C.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.**

In addition to Proficient, the **Advanced** student is able to prove theorems about lines and angles using multiple representations (constructions, analytic geometry, theorems, etc.) and/or analyze and critique proofs written by others by verifying the logic.

The **Proficient** student is able to prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

The **Basic** student is able to:

- Give an informal explanation of the relationship of pairs of lines and/or angles. **AND/OR**
- Complete a partial proof by filling in the blanks when given either a statement or a reason.

The **Below Basic** student may be able to identify:

- Vertical angles are congruent;
- When a transversal crosses parallel lines, alternate interior angles are congruent; **AND**
- Points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

**G.CO.C.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.**

In addition to Proficient, the **Advanced** student is able to:

- Prove theorems about triangles using multiple representations (constructions, analytic geometry, theorems, etc.) **AND/OR**
- Analyze and critique proofs written by others by verifying the logic.

The **Proficient** student is able to prove theorems about triangles. Theorems include: measure of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

The **Basic** student is able to:

- Give informal explanations of triangle proofs. **AND/OR**
- Complete a partial proof by filling in the blanks when given either a statement or a reason.

Theorems include: measure of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

The **Below Basic** student may be able to identify triangle theorems. Theorems include: measure of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

**G.CO.C.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.**

In addition to Proficient, the **Advanced** student is able to, using theorems about parallelograms and triangles, prove other conjectures about figures that are compositions of parallelograms and/or triangles and/or circles.

The **Proficient** student is able to prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

The **Basic** student is able to informally prove, (including by measurement or inspection) theorems about parallelograms, **and**:

- Complete proofs of theorems about parallelograms, **AND/OR**
- Order the steps of a proof of theorems about parallelograms.

The **Below Basic** student may be able to, given a proof and figure:

- Identify opposite and consecutive sides or angles in a figure. **OR**
- Identify and define congruent figures using symbols or given notation. **OR**
- Identify and define parallel lines using symbols or given notation. **OR**
- Identify and define perpendicular lines symbols or given notation. **OR**
- Define supplementary angles. **OR**
- Draw or identify diagonals of a polygon.

## Make geometric constructions.

**G.CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.**

In addition to Proficient, the **Advanced** student is able to use formal geometric constructions to prove theorems about parallelograms, circles, and triangles or prove that the formal geometric construction copies a line segment; copies an angle; bisects a segment; bisects an angle; constructs perpendicular lines, including perpendicular bisectors of a line segment; constructing a line parallel to a given line through a point not on the line.

The **Proficient** student is able to make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

The **Basic** student is able to make some, but not all, formal geometric constructions using at least one tool or method:

- Copying a segment.
- Copying an angle.
- Bisecting a segment.
- Bisecting an angle.
- Constructing perpendicular lines, including the perpendicular bisector of a line segment.
- Constructing a line parallel to a given line through a point not on the line.

The **Below Basic** student may be able to make some, but not all, formal geometric constructions using at least one tool or method with assistance:

- Copying a segment.
- Copying an angle.
- Bisecting a segment.

- Bisecting an angle.
- Constructing perpendicular lines, including the perpendicular bisector of a line segment.
- Constructing a line parallel to a given line through a point not on the line.

**G.CO.D.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.**

In addition to Proficient, the **Advanced** student is able to construct an equilateral triangle, a square, and other regular polygons (e.g., pentagon, hexagon, octagon) inscribed in a circle and justify the tools and techniques used.
The **Proficient** student is able to construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
The **Basic** student is able to construct an equilateral triangle, a square, or a regular hexagon inscribed in a circle.
The **Below Basic** student may be able to construct an equilateral triangle, a square, or a regular hexagon inscribed in a circle with assistance (e.g., video, ordering completed steps of a construction, etc.).

## SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY

### Understand similarity in terms of similarity transformations.

**G.SRT.E.1 Verify heuristically the properties of dilations given by a center and a scale factor.**

**G.SRT.E.1A** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
**G.SRT.E.1B** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

In addition to Proficient, the **Advanced** student is able to prove the properties of dilations given by a center and a scale factor.

A. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
B. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

The **Proficient** student is able to understand similarity in terms of similarity transformations. Verify heuristically the properties of dilations given by a center and a scale factor. (A heuristic approach is an approach to problem solving or discovery that employs practical method that is not guaranteed to be optimal or perfect, but is sufficient for the immediate goals.)

A. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
B. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

The **Basic** student is able to demonstrate that two figures are similar using the given center of dilation and the scale factor.
The **Below Basic** student may be able to recognize that two figures resulting from a dilation are similar.

**G.SRT.E.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.**

In addition to Proficient, the **Advanced** student is able to, given two figures, use the definition of similarity in terms of similarity transformations and other theorems to prove that the figures are similar; paying particular attention to the equality of corresponding angle pairs and the proportionality of corresponding side pairs.
The **Proficient** student is able to, given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

The **Basic** student is able to, given two similar figures, demonstrate that corresponding angle pairs are congruent and that corresponding side pairs are in proportion using transformations and/or measurement.
The **Below Basic** student may be able to, given two similar figures, identify congruent corresponding angle pairs and corresponding side pairs.

**G.SRT.E.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.**

In addition to Proficient, the **Advanced** student is able to prove that two triangles are similar or dissimilar using the AA criterion for two similar triangles or prove the AA criterion.
The **Proficient** student is able to use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
The **Basic** student is able to, given two triangles, determine similarity and dissimilarity using the AA criterion.
The **Below Basic** student may be able to, given two similar triangles identify the two pairs of angles that are congruent.

## Prove theorems involving similarity.

**G.SRT.F.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.**

In addition to Proficient, the **Advanced** student is able to prove theorems about other regular figures by making generalizations of triangle proofs and theorems.
The **Proficient** student is able to prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
The **Basic** student is able to informally prove, (including by measurement or inspection) theorems about triangles, **and:**

- Complete proofs of theorems about triangles. **AND/OR**
- Order the steps of a proof of theorems about triangles.

Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
The **Below Basic** student may be able to, with assistance:

- Informally prove, (including by measurement or inspection) theorems about triangles. **OR**
- Complete proofs of theorems about triangles. **OR**
- Order the steps of a proof of theorems about triangles.

Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

**G.SRT.F.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.**

In addition to Proficient, the **Advanced** student is able to use congruence and similarity criteria for triangles to solve problems and explain why a geometric figure has been incorrectly deemed congruent or similar.
The **Proficient** student is able to use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
The **Basic** student is able to use congruence or similarity criteria for triangles to solve problems or to prove relationships in geometric figures.
The **Below Basic** student may be able to, when given specific information about similarity or congruent triangles, solve problems or identify the given relationship for the triangles.

## Define trigonometric ratios and solve problems involving right triangles.

**G.SRT.G.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.**

In addition to Proficient, the **Advanced** student is able to using data taken from several pairs of similar right triangles, establish generalities about the sides of right triangles and their relationship to the acute angles of said triangles.
The **Proficient** student is able to understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
The **Basic** student is able to demonstrate the ability to correctly orient two similar right triangles in order to identify the relationship between sides and angles.
The **Below Basic** student may be able to identify (verbally or in writing) the relationship between sides and angles when given two similar right triangles, each similarly oriented.

**G.SRT.G.7 Explain and use the relationship between the sine and cosine of complementary angles.**

In addition to Proficient, the **Advanced** student is able to use data about side lengths of several right triangles and the properties of similar triangles, derive generalities about the sine and cosine relationships found.
The **Proficient** student is able to explain and use the relationship between the sine and cosine of complementary angles.
The **Basic** student is able to use the relationship between the sine and cosine of complementary angles.
The **Below Basic** student may be able to use the sine and cosine relationships with assistance.

**G.SRT.G.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.**

In addition to Proficient, the **Advanced** student is able to use trigonometric ratios and the Pythagorean Theorem to solve right triangles and verify proposed solutions in applied problems.
The **Proficient** student is able to use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
The **Basic** student is able to solve right triangles in applied problems when given the trigonometric ratios and the Pythagorean Theorem.
The **Below Basic** student may be able to identify parts of a given right triangle figure that correspond to an applied problem for use in the formula when given trigonometric ratios and/or the Pythagorean Theorem.

## Apply trigonometry to general triangles.

**G.SRT.H.9 (+) Derive the formula $A = \frac{1}{2}ab\ sin(c)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.**

(+) In addition to Proficient, the **Advanced** student is able to:

- Explain if the formula, $A = \frac{1}{2}ab\ sin(C)$ would be appropriate for use in a compound figure and if possible, use the formula to determine the area of the compound figure. **OR**
- Use the area formula in a novel way.

(+) The **Proficient** student is able to derive the formula $A = \frac{1}{2}ab\ sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

(+) The **Basic** student is able to complete the derivation of the formula $A = \frac{1}{2}ab\ sin(C)$ by choosing appropriate steps from a provided list or correctly ordering the steps of a completed derivation.

(+) The **Below Basic** student may be able to use $A = \frac{1}{2}ab\ sin(C)$ to find the area of a triangle when provided the formula and a labeled figure.

**G.SRT.H.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.**

(+) In addition to Proficient, the **Advanced** student is able to:

- Demonstrate instances when using the Law of Sines or the Law of Cosines would not be appropriate. **OR**
- Detect errors in the work of others.

(+) The **Proficient** student is able to prove the Law of Sines and the Law of Cosines, and use them to solve problems.

(+) The **Basic** student is able to use the Law of Sines and the Law of Cosines to solve problems when provided with the laws and complete proofs of the Law of Sines and the Law of Cosines by:

- Choosing appropriate steps from a provided list. **OR**
- Correctly ordering the steps of completed proofs.

(+) The **Below Basic** student may be able to use the Law of Sines and the Law of Cosines to solve problems when provided with the laws **or** complete proofs of the Law of Sines and the Law of Cosines by:

- Choosing appropriate steps from a provided list. **OR**
- Correctly ordering the steps of completed proofs.

**G.SRT.H.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).**

(+) In addition to Proficient, the **Advanced** student is able to:

- Demonstrate instances when using the Law of Sines or Law of Cosines would not be appropriate. **OR**
- Detect errors in the work of others.

(+) The **Proficient** student is able to understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

(+) The **Basic** student is able to use the Law of Sines and the Law of Cosines to solve problems when provided with the laws.

(+) The **Below Basic** student may be able to use the Law of Sines and Law of Cosines to solve problems when provided with the laws when given assistance.

## CIRCLES

### Understand and apply theorems about circles.

**G.C.I.1 Prove that all circles are similar.**

In addition to Proficient, the **Advanced** student is able to:

- Extend the proof that all circles are similar to other appropriate curvilinear figures. **OR**
- Detect errors in the work of others

The **Proficient** student is able to prove that all circles are similar.

The **Basic** student is able to explain the similar relationship between two circles.

The **Below Basic** student may be able to recognize that two circles are similar and describe in simple terms the relationship verbally or in writing.

**G.C.I.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.**

In addition to Proficient, the **Advanced** student is able to apply relationships among inscribed angles, radii, and chords to solve real-world problems. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

The **Proficient** student is able to identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

The **Basic** student is able to identify and describe some relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

The **Below Basic** student may be able to identify or describe some relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

**G.C.I.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.**

In addition to Proficient, the **Advanced** student is able to:

- Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for regular polygons inscribed in a circle. **OR**
- Solve real-world problems using the properties described above.

The **Proficient** student is able to construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

The **Basic** student is able to construct the inscribed and circumscribed circles of a triangle, and complete proofs of properties of angles for a quadrilateral inscribed in a circle by:

- Choosing appropriate steps from a provided list. **OR**
- Correctly ordering the steps of completed proofs.

The **Below Basic** student may be able to construct the inscribed and circumscribed circles of a triangle, **or** complete proofs of properties of angles for a quadrilateral inscribed in a circle by:

- Choosing appropriate steps from a provided list. **OR**
- Correctly ordering the steps of completed proofs.

**G.C.I.4 (+) Construct a tangent line from a point outside a given circle to the circle.**

(+) In addition to Proficient, the **Advanced** student is able to:

- Prove the construction of a tangent line from a point outside of a circle to a point of tangency is unique. **OR**
- Apply this concept to solve a real-world problem.

(+) The **Proficient** student is able to construct a tangent line from a point outside a given circle to the circle.

(+) The **Basic** student is able to construct a tangent line from a point outside a given circle to the circle with assistance.

(+) The **Below Basic** student may be able to distinguish between tangent lines from a point outside a given circle to the circle, and lines from the same or a different point outside a given circle to the circle, that are not tangent to a circle (i.e., identify tangency).

### Find arc lengths and areas of sectors of circles.

**G.C.J.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.**

In addition to Proficient, the **Advanced** student is able to find the constant of proportionality by comparing arc lengths and sector areas of angles with differing radii measurements. (This is an extension of G.SRT.1 – properties of dilations).

The **Proficient** student is able to derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

The **Basic** student is able to recognize a sector as a part of the whole measure of the area of a circle with the same radius. Find the area of that sector using the formula for the area of a sector and find the arc length of the segment created by the sector.

The **Below Basic** student may be able to:

- Recognize a sector as a part of the whole measure of the area of a circle with the same radius. **OR**
- Find the area of that sector using the formula for the area of a sector and find the arc length of segment created by the sector.

## EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS

### Translate between the geometric description and the equation for a conic section.

**G.GPE.K.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.**

In addition to Proficient, the **Advanced** student is able to use algebraic techniques to draw connections between distance formula, Pythagorean Theorem, completing the square, and transformations of functions to write the equation of a circle and to develop logical arguments for the standard form of a circle.

The **Proficient** student is able to derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

The **Basic** student is able to derive the equation of a circle of given center and radius using the Pythagorean Theorem or complete the square to find the center and radius of a circle given by an equation.

The **Below Basic** student may be able to derive the equation of a circle of given center and radius using the Pythagorean Theorem with assistance or complete the square to find the center and radius of a circle given by an equation with assistance.

**G.GPE.K.2 (+) Derive the equation of a parabola given a focus and directrix.**

(+) In addition to Proficient, the **Advanced** student is able to draw connections between standard form and conic form of parabola equations **or** detect errors in others' derivation of equations.

(+) The **Proficient** student is able to derive the equation of a parabola given a focus and directrix.

(+) The **Basic** student is able to derive the equation of a parabola given a focus and directrix by:

- Choosing appropriate steps from a provided list. **OR**
- Correctly ordering the steps of completed proofs.

(+) The **Below Basic** student may be able to, with assistance, derive the equation of a parabola given a focus and directrix by:

- Choosing appropriate steps from a provided list. **OR**
- Correctly ordering the steps of completed proofs.

**G.GPE.K.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.**

(+) In addition to Proficient, the **Advanced** student is able to:

- Draw connections between the standard forms and the conic forms of ellipses and hyperbolas. **OR**
- Detect errors in others' derivations of said equations.

(+) The **Proficient** student is able to derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

(+) The **Basic** student is able to derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant by:

- Choosing appropriate steps from a provided list. **OR**
- Correctly ordering the steps of completed proofs.

(+) The **Below Basic** student may be able to, with assistance, derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant by:

- Choosing appropriate steps from a provided list. **OR**
- Correctly ordering the steps of completed proofs.

## Use coordinates to prove simple geometric theorems algebraically.

**G.GPE.L.4 Use coordinates to prove simple geometric theorems algebraically.**

In addition to Proficient, the **Advanced** student is able to:

- Use coordinates to prove complex geometric theorems. **OR**
- Detect errors in the proofs of others.

The **Proficient** student is able to use coordinates to prove simple geometric theorems algebraically.

The **Basic** student is able to use coordinates to prove simple geometric theorems algebraically by:

- Choosing appropriate steps from a provided list. **OR**
- Correctly ordering the steps of completed proofs.

The **Below Basic** student may be able to, with assistance, use coordinates to prove simple geometric theorems algebraically by:

- Choosing appropriate steps from a provided list. **OR**
- Correctly ordering the steps of completed proofs.

**G.GPE.L.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).**

In addition to Proficient, the **Advanced** student is able to:

- Make observations and develop logical arguments about the relationship of lines found in real-world contexts (parallel and perpendicular).
- Analyze and critique the work of others in using and proving slope criteria.

The **Proficient** student is able to prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

The **Basic** student is able to informally prove the slope criteria for parallel or perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

The **Below Basic** student may be able to verify perpendicular and parallel lines when given different representations (e.g., graphs, tables, equations) and use them to solve geometric problems.

**G.GPE.L.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.**

In addition to Proficient, the **Advanced** student is able to explore and draw conclusions about patterns found among directed line segments or geometric figures of different dimensions and different ratios (e.g., patterns in the coordinates, patterns in actual lengths, patterns when changing units).
The **Proficient** student is able to find the point on a directed line segment between two given points that partitions the segment in a given ratio.
The **Basic** student is able to find the point on a directed line segment between two given points that partitions the segment in a given common unit ratio (e.g., $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$).
The **Below Basic** student may be able to given a directed line segment, identify the point between two given points for common parts and wholes in a whole ratio using unit ratios (e.g., $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$).

**G.GPE.L.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, (e.g., using the distance formula).**

In addition to Proficient, the **Advanced** student is able to use coordinates to compute areas of polygons and verify the technique using alternative methods.
The **Proficient** student is able to use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).
The **Basic** student is able to use coordinates to compute perimeters and areas of triangles and rectangles.
The **Below Basic** student may be able to use coordinates to compute perimeters or areas of right triangles or rectangles.

## GEOMETRIC MEASUREMENT AND DIMENSION

### Explain volume formulas and use them to solve problems.

**G.GMD.M.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.**

In addition to Proficient, the **Advanced** student is able to:

- Critique the method and verify the logic used by others. **OR**
- Use dissection arguments, Cavalieri's principle, and informal limit arguments to find the area or volume of real-world irregular figures (e.g., horseshoe, hand, putting green, area under a curve).

The **Proficient** student is able to give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
The **Basic** student is able to, given a figure, identify the unit shapes that could be used to determine the area and use the sum of the parts to determine a method that approximates the area. Given a three dimensional figure, identify the unit shapes that could be used to determine the volume and use the sum of the parts to determine a method that approximates the volume.
The **Below Basic** student may be able to, given a figure, identify the unit shapes that could be used to determine the area. Given a three dimensional figure, identify the unit shapes that could be used to determine the volume.

**G.GMD.M.2 (+) Give an informal argument using Cavalieri's Principle for the formulas for the volume of a sphere and other solid figures.**

(+) In addition to Proficient, the **Advanced** student is able to give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures and explain their use in real-world situations.
(+) The **Proficient** student is able to give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
(+) The **Basic** student is able to, given several sliced solid figures with dimensions labeled, give an informal argument to explain why some figures have equal volumes.
(+) The **Below Basic** student may be able to, given several sliced solid figures with dimensions labeled, identify the figures that have equal volumes.

**G.GMD.M.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.**

In addition to Proficient, the **Advanced** student is able to use volume formulas for cylinders, pyramids, cones, and spheres to solve problems with compound shapes built from cylinders, pyramids, cones, and/or spheres in real-world contexts.
The **Proficient** student is able to use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
The **Basic** student is able to use given volume formulas and shapes with all of the dimensions labeled for cylinders, pyramids, cones, and spheres to solve problems.
The **Below Basic** student may be able to use given volume formulas and shapes with all of the dimensions labeled for cylinders, pyramids, cones, or spheres to solve problems.

## Visualize relationships between two-dimensional and three-dimensional objects.

**G.GMD.M.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional object.**

In addition to Proficient, the **Advanced** student is able to:

- Use an irregular two-dimensional slice and generate the three dimensional object created from rotating it about an axis. **OR**
- Explore the slicing of commonly shaped solids and analyze the patterns you find when making different slices. **OR**
- Show where to slice a cube or cylinder to get a minimum and maximum number of sides of the two dimensional cross-sections.

The **Proficient** student is able to identify the shapes of two dimensional cross-sections of three dimensional objects, and identify three dimensional objects generated by rotations of two-dimensional objects.
The **Basic** student is able to identify the shapes of two dimensional cross-sections of three dimensional objects, or identify three dimensional objects generated by rotations of two-dimensional objects.
The **Below Basic** student may be able to identify the shapes of two dimensional cross-sections of three dimensional objects when sliced horizontally or vertically, or identifies three dimensional objects generated by rotations about a horizontal or vertical edge of two-dimensional objects.

## MODELING WITH GEOMETRY

### Apply geometric concepts in modeling situations.

**G.MG.O.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).**

In addition to Proficient, the **Advanced** student is able to combine three-dimensional shapes to create a real-world object and estimate the volume.
The **Proficient** student is able to use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
The **Basic** student is able to choose the appropriate combination of geometric shapes to describe a specified object.
The **Below Basic** student may be able to choose the appropriate geometric shape to describe a specified object.

**G.MG.O.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).**

In addition to Proficient, the **Advanced** student is able to critique the reasoning of others/self and apply concepts of density (density = mass/volume) based on area and volume in modeling situations and compare results to real-world data, if available, to make adjustments to estimation methods.
The **Proficient** student is able to apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
The **Basic** student is able to, given two different unit rates of density based on area or volume, compare totals when given the overall area or volume (e.g., given persons per square mile estimate the total for each given area and compare them, given BTUs per cubic foot estimate the total for more than one larger volume and compare them).
The **Below Basic** student may be able to, given a unit rate of density based on area or volume, find an estimated total in a modeling situation (e.g., given persons per square mile estimate the total for a given area, given BTUs per cubic foot estimate the total for a larger volume).

**G.MG.O.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).**

In addition to Proficient, the **Advanced** student is able to design an object or structure to satisfy physical constraints and minimize cost and justify the reasoning.
The **Proficient** student is able to apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
The **Basic** student is able to, given volume and surface area formulas, (prisms, pyramids, and spheres) determine the amount of material required to create a structure with specific physical constraints.
The **Below Basic** student may be able to identify which geometric attribute(s) need(s) to be calculated and use given volume and surface area formulas, (prisms, pyramids, and spheres) to determine the amount of material required to create a structure with specific physical constraints.

## INTERPRETING CATEGORICAL AND QUANTITATIVE DATA

### Summarize, represent, and interpret data on a single count or measurement variable.

**S.ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots) by hand or using technology.**

In addition to Proficient, the **Advanced** student is able to compare and contrast the different data representations (dot plots, histograms, and box plots) to determine what information can be gleaned or lost from each and justify the most appropriate representation to use.
The **Proficient** student is able to represent data with plots on the real number line (dot plots, histograms, and box plots) by hand or using technology.
The **Basic** student is able to represent data with plots on the real number line (using dot plots, histograms, or box plots).
The **Below Basic** student may be able to represent data with plots on the real number line (using a specified representation: dot plots and/or histograms and/or box plots) and given a pictorial or verbal example of each type that is to be created.

**S.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.**

In addition to Proficient, the **Advanced** student is able to find the appropriate visual representation and justify the appropriate measure of center and spread, given two or more sets of data.
The **Proficient** student is able to use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
The **Basic** student is inconsistently able to use appropriate terminology to describe similarities and differences when comparing center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
The **Below Basic** student may be able to informally compare the similarities and differences when comparing center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

**S.ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).**

In addition to Proficient, the **Advanced** student is able to given different shapes, context, and statistics for sets of data, discern and predict the differences caused by omission or inclusion of extreme data points (outliers) in data sets, including those with uniform or near uniform values.
The **Proficient** student is able to interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
The **Basic** student is able to interpret differences in any two of the following: shape, center, or spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
The **Below Basic** student may be able to when comparing different data sets using the same representation, Identify the impact of extreme data points (outliers) on the shape (skewed, symmetrical, or constant), center (mean or median), or spread (interquartile range or standard deviation).

**S.ID.A.4 (+) Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use the Empirical Rule, calculators, spreadsheets, and/or tables to estimate areas under the normal curve.**

(+) In addition to Proficient, the **Advanced** student is able to, using the mean, standard deviation, other statistics, and visual representations of several data sets, compare the visual representations to determine and justify the appropriateness of fitting to a normal curve to estimate population percentages.
(+) The **Proficient** student is able to use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use the Empirical Rule, calculators, spreadsheets, and/or tables to estimate areas under the normal curve.
(+) The **Basic** student is able to use the mean and standard deviation of a data set to attempt to fit it to a normal distribution and to estimate population percentages, not recognizing when such procedures might not be appropriate. Use the Empirical Rule, calculators, spreadsheets, and/or tables to make population estimates.
(+) The **Below Basic** student may be able to, given a normal distribution, estimate population percentages using the Empirical Rule, calculators, spreadsheets, and/or tables.

**S.ID.B.5 (+) Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations in the data, and use inferential statistical techniques to show association.**

(+) In addition to Proficient, the **Advanced** student is able to identify real-world problems where chi-square analysis (i.e., goodness of fit, homogeneity, and independence) would be appropriate and use chi-square analysis to solve these problems.
(+) The **Proficient** student is able to summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations in the data, and use inferential statistical techniques to show association.
(+) The **Basic** student is able to summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and/or conditional relative frequencies).
(+) The **Below Basic** student does not meet the basic performance level.

**S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.**

**S.ID.B.6A** Use a function to describe data trends to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
**S.ID.B.6B** (+) Informally assess the fit of a function by plotting and analyzing residuals.
**S.ID.B.6C** Using technology, fit a least squares linear regression function for a scatter plot that suggests a linear association.

In addition to Proficient, the **Advanced** student is able to:

- Determine the best model and justify by describing the pros and cons of the data representation for two quantitative variables on a scatter plot using alternative linear, quadratic, and exponential models. **AND**
- Determine the effect of removing extreme values (outliers) on the model. Make an argument for or against removing extreme values (outliers). **AND**
- Discuss appropriate use of the model to make predictions while attending to precision (correct data entry errors).

The **Proficient** student is able to represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

A. Use a function to describe data trends to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
B. (+) Informally assess the fit of a function by plotting and analyzing residuals.
C. Using technology, fit a least squares linear regression function for a scatter plot that suggests a linear association.

The **Basic** student is able to:

- Given two models (linear, quadratic, or exponential) determine which model best represents the data by informally assessing the fit of a function by plotting and/or analyzing residuals. **OR**
- Given a data set, use technology to create a scatter plot and the least squares regression function.

The **Below Basic** student may be able to represent data on two quantitative variables on a scatter plot and informally determine if a linear, quadratic, or exponential is a best fit.

## Interpret linear models.

**S.ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.**

In addition to Proficient, the **Advanced** student is able to:

- Make predictions using the rate of change and the constant term of a linear model in the context of the data. Determine and explain when extrapolation is appropriate or inappropriate. **OR**
- Identify a data source, formulate questions about the data, and explain in context how the slope and intercept would be interpreted.

The **Proficient** student is able to interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
The **Basic** student is able to determine the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
The **Below Basic** student may be able to, given the slope (rate of change) and the intercept (constant term) of a linear model, locate these in a scatter plot of the same data.

**S.ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.**

In addition to Proficient, the **Advanced** student is able to compare different scatter plots with the same correlation coefficient to determine if a linear model best fits the data by examining shape and statistics and justify reasoning.
The **Proficient** student is able to compute (using technology) and interpret the correlation coefficient of a linear fit.
The **Basic** student is able to compute (using technology) or interpret the correlation coefficient of a linear fit.
The **Below Basic** student may be able to compute (using technology) or interpret the correlation coefficient of a linear fit with the assistance of written or pictorial guided steps or video.

**S.ID.C.9 Distinguish between correlation and causation.**

In addition to Proficient, the **Advanced** student is able to:

- Support or refute claims of causation from a real-world example (e.g., newspaper, website) with the understanding that a strong correlation does not imply causation. **OR**
- Research or create two sets of data and their context to demonstrate how correlation and causation could be confused. Explain the reasons for confusion.

The **Proficient** student is able to distinguish between correlation and causation.
The **Basic** student is able to identify the existence or nonexistence of causation in the context of a correlated problem.
The **Below Basic** student may be able to, when provided with two disparate examples, determine which demonstrates correlation.

# MAKING INFERENCES AND JUSTIFYING CONCLUSIONS

## Understand and evaluate random processes underlying statistical experiments.

**S.IC.D.1 (+) Understand statistics as a process for making inferences about population parameters based on a random sample from that population.**

(+) In addition to Proficient, the **Advanced** student is able to select a random sample from a real-world population and use statistics to make appropriate inferences.
(+) The **Proficient** student is able to use statistics as a process for making inferences about population parameters based on a random sample from that population.
(+) The **Basic** student is able to draw logical conclusions about a population when provided with descriptive statistics from a random sample from that population.
(+) The **Below Basic** student does not meet the basic performance level.

**S.IC.D.2 (+) Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.**

(+) In addition to Proficient, the **Advanced** student is able to generate or estimate a model consistent with results from a given data-generating process, e.g., using simulation.
(+) The **Proficient** student is able to decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
(+) The **Basic** student is able to match a plot for data from a specified real-world situation with a given model.
(+) The **Below Basic** student does not meet the basic performance level.

## Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

**S.IC.E.3 (+) Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.**

(+) In addition to Proficient, the **Advanced** student is able to draw multiple random samples to complete a survey, experiment, or observational study. Compare and discuss the results.
(+) The **Proficient** student is able to recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
(+) The **Basic** student is able to explain how results can be biased if the sample is not randomly selected, e.g., a convenience sample vs. a random sample.
(+) The **Below Basic** student does not meet the basic performance level.

**S.IC.E.4 (+) Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.**

(+) In addition to Proficient, the **Advanced** student is able to develop a confidence interval for the population mean or proportion using the data from the sample survey and relate it to the margin of error from the simulation.
(+) The **Proficient** student is able to use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
(+) The **Basic** student is able to use data from a sample survey to estimate a population mean or proportion and use the formula to calculate the margin of error.
(+) The **Below Basic** student does not meet the basic performance level.

**S.IC.E.5 (+) Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.**

(+) In addition to Proficient, the **Advanced** student is able to do a statistical analysis (i.e., t-test) of the data from a randomized experiment to compare two treatments. Report results to determine if the differences between the parameters are significant.
(+) The **Proficient** student is able to use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
(+) The **Basic** student is able to, given a plot comparing the two treatments from a randomized experiment, construct a logical argument about whether or not the parameters (proportion or mean) would be different.
(+) The **Below Basic** student does not meet the basic performance level.

**S.IC.E.6 (+) Evaluate reports based on data.**

(+) In addition to Proficient, the **Advanced** student is able to evaluate a report discussing the sampling technique, the data collection instruments, the assumptions of the statistical analysis used, the data analysis, and the accuracy of the conclusions drawn.
(+) The **Proficient** student is able to (+) evaluate reports based on data.
(+) The **Basic** student is able to evaluate reports by identifying the type of sampling done and comment on its appropriateness. Discuss if there is data provided to support the conclusions.
(+) The **Below Basic** student does not meet the basic performance level.

## CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY

### Understand independence and conditional probability and use them to interpret data.

**S.CP.F.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").**

In addition to Proficient, the **Advanced** student is able to:

- Compare at least two different representations (e.g., Venn Diagram, two-way table, set notation, verbal description) of events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **OR**
- Develop questions that can be answered using unions, intersections, or complements.

The **Proficient** student is able to describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
The **Basic** student is able to describe events as subsets using three of the four characteristics:

- Outcomes.
- Unions.
- Intersection.
- Complements.

The **Below Basic** student may be able to describe events as subsets using two of the four characteristics:

- Outcomes.
- Unions.

- Intersection.
- Complements.

**S.CP.F.2 (+) Understand that two events $A$ and $B$ are independent if the probability of $A$ and $B$ occurring together is the product of their probabilities, and use this characterization to determine if they are independent.**

(+) In addition to Proficient, the **Advanced** student is able to identify real-world situations where $P(A)$ and $P(B)$ can be used to determine if the events $A$ and $B$ are independent by deriving the probabilities $P(A)$, $P(B)$, $P(A \text{ and } B)$ and interpret results.
(+) The **Proficient** student is able to understand that two events $A$ and $B$ are independent if the probability of $A$ and $B$ occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
(+) The **Basic** student is able to determine if events $A$ and $B$ are independent when given probabilities $P(A)$, $P(B)$, and $P(A \text{ and } B)$.
(+) The **Below Basic** student may be able to does not meet the basic performance level.

**S.CP.F.3 (+) Understand the conditional probability of $A$ given $B$ as $P(A \text{ and } B)/P(B)$, and interpret independence of $A$ and $B$ as saying that the conditional probability of $A$ given $B$ is the same as the probability of $A$, and the conditional probability of $B$ given $A$ is the same as the probability of $B$.**

(+) In addition to Proficient, the **Advanced** student is able to identify real-world situations where $P(A)$ and $P(B)$ can be used to determine if the events $A$ and $B$ are independent by deriving the probabilities $P(A)$, $P(B)$, $P(A|B)$, $P(B|A)$, and $P(A \text{ and } B)$ and interpret results.
(+) The **Proficient** student is able to (+) understand the conditional probability of $A$ given $B$ as $P(A \text{ and } B)/P(B)$, and interpret independence of $A$ and $B$ as saying that the conditional probability of $A$ given $B$ is the same as the probability of $A$, and the conditional probability of $B$ given $A$ is the same as the probability of $B$.
(+) The **Basic** student is able to determine if events $A$ & $B$ are independent when given probabilities $P(A)$, $P(B)$, $P(A|B)$, $P(B|A)$, and $P(A \text{ and } B)$.
(+) The **Below Basic** student does not meet the basic performance level.

**S.CP.F.4 (+) Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.**

(+) In addition to Proficient, the **Advanced** student is able to identify a real-world situation and collect data that is appropriate for constructing a two-way frequency table. Construct and interpret the two-way frequency table of data when two categories are associated with each object being classified. Analyze and describe the results.
(+) The **Proficient** student is able to construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
(+) The **Basic** student is able to interpret a given two-way frequency table of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
(+) The **Below Basic** student does not meet the basic performance level.

**S.CP.F.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.**

In addition to Proficient, the **Advanced** student is able to identify a real-world situation that uses independence and identify a real-world situation that uses dependence.
The **Proficient** student is able to recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
The **Basic** student is able to recognize the concepts of conditional probability or independence in everyday language and everyday situations.

The **Below Basic** student may be able to, with assistance, recognize the concepts of conditional probability or independence in everyday language and everyday situations.

## Use the rules of probability to compute probabilities of compound events in a uniform probability model.

**S.CP.G.6 (+) Find the conditional probability of $A$ given $B$ as the fraction of $B$'s outcomes that also belong to $A$, and interpret the answer in terms of the model.**

(+) In addition to Proficient, the **Advanced** student is able to explain why $P(A|B)$ is different than $P(B|A)$ in a real-world situation.
(+) The **Proficient** student is able to find the conditional probability of $A$ given $B$ as the fraction of $B$'s outcomes that also belong to $A$, and interpret the answer in terms of the model.
(+) The **Basic** student is able to calculate conditional probability of $A$ given $B$ when given probabilities $P(A)$, $P(B)$, $P(A|B)$, $P(B|A)$, and $P(A \text{ and } B)$.
(+) The **Below Basic** student does not meet the basic performance level.

**S.CP.G.7 (+) Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.**

(+) In addition to Proficient, the **Advanced** student is able to identify real-word situations where the addition rule would apply, derive the appropriate probabilities, solve the problem, and interpret the results.
(+) The **Proficient** student is able to apply the addition rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
(+) The **Basic** student is able to apply the addition rule and interpret results when given probabilities $P(A)$, $P(B)$, and $P(A \text{ and } B)$.
(+) The **Below Basic** student does not meet the basic performance level.

**S.CP.G.8 (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = [P(A)]x[P(B|A)] = [P(B)]x[P(A|B)]$, and interpret the answer in terms of the model.**

(+) In addition to Proficient, the **Advanced** student is able to identify real-word situations where the multiplication rule would apply, derive the appropriate probabilities, solve the problem and interpret the results.
(+) The **Proficient** student is able to apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = [P(A)]x\,[P(B|A)] = [P(B)]x[P(A|B)]$, and interpret the answer in terms of the model.
(+) The **Basic** student is able to apply the multiplication rule and interpret results when given probabilities $P(A)$, $P(B)$, $P(A|B)$, and $P(B|A)$.
(+) The **Below Basic** student does not meet the basic performance level.

**S.CP.G.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.**

(+) In addition to Proficient, the **Advanced** student is able to identify a real-world problem that requires permutations and combinations to compute the probability of a compound event, derive the required values, solve the problem, and interpret the results.
(+) The **Proficient** student is able to use permutations and combinations to compute probabilities of compound events and solve problems.
(+) The **Basic** student is able to determine the probability of a compound event when given the values of the permutations and the combinations applicable to the problem.
(+) The **Below Basic** student does not meet the basic performance level.

## USE PROBABILITY TO MAKE DECISIONS

### Calculate expected values and use them to solve problems.

**S.MD.H.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.**

(+) In addition to Proficient, the **Advanced** student is able to construct a statistical experiment by identifying an appropriate random variable, listing the events in the sample space, calculating the probability distribution, and constructing the appropriate graphical display. Interpret the results.
(+) The **Proficient** student is able to define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
(+) The **Basic** student is able to, given a random variable for a quantity of interest and the numerical value for each event in the sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
(+) The **Below Basic** student does not meet the basic performance level.

**S.MD.H.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.**

(+) In addition to Proficient, the **Advanced** student is able to gather data for a real-world situation where expected value could be used to make an advantageous decision (e.g., lottery ticket, card game, dice game, investments). Calculate the expected value and explain or support the decision.
(+) The **Proficient** student is able to calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
(+) The **Basic** student is able to calculate expected value when given the formula $E(X) = \Sigma(X_i * P(X_i))$ and a table with $X_i$ and $P(X_i)$.
(+) The **Below Basic** student does not meet the basic performance level.

**S.MD.H.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.**

(+) In addition to Proficient, the **Advanced** student is able to construct a statistical experiment with a random variable and develop a table of $X_i$, $P(X_i)$ values, and determine the expected value. Interpret the results.
(+) The **Proficient** student is able to develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.
(+) The **Basic** student is able to develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated.
(+) The **Below Basic** student does not meet the basic performance level.

**S.MD.H.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.**

(+) In addition to Proficient, the **Advanced** student is able to simulate a random process by defining the sample space and developing the probability distribution for a random variable. Calculate the expected value and interpret the results.
(+) The **Proficient** student is able to develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
(+) The **Basic** student is able to complete a probability distribution for a random variable from a simple experiment (e.g., a coin flip, tossing a die, drawing a card) defined for a sample space in which probabilities are assigned empirically.

(+) The **Below Basic** student does not meet the basic performance level.

## Use probability to evaluate outcomes of decisions.

**S.MD.I.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.**

**S.MD.I.5A** Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.

**S.MD.I.5B** Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.

(+) In addition to Proficient, the **Advanced** student is able to gather data to make the advantageous decision for a real-world situation. Explain reasoning for the decision.

(+) The **Proficient** student is able to weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

A. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.

B. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.

(+) The **Basic** student is able to weigh the possible outcomes of a decision by assigning probabilities to payoff values and/or finding expected values. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast- food restaurant.

(+) The **Below Basic** student does not meet the basic performance level.

## Calculate expected values and use them to solve problems.

**S.MD.J.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).**

(+) In addition to Proficient, the **Advanced** student is able to identify real-world situations where probability could be used to make fair and unfair decisions. Explain the reasoning.

(+) The **Proficient** student is able to use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

(+) The **Basic** student is able to compare given probability distributions. State which distribution would result in a fair decision.

(+) The **Below Basic** student does not meet the basic performance level.

**S.MD.J.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).**

(+) In addition to Proficient, the **Advanced** student is able to analyze decisions and strategies using probability concepts, identify the advantages and disadvantages of the possible decisions, and justify the best choice (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

(+) The **Proficient** student is able to analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

(+) The **Basic** student is able to analyze decisions and strategies using probability concepts when given the probabilities (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

(+) The **Below Basic** student does not meet the basic performance level.